

Attempt **FOUR** Questions, selecting at least **ONE** from each section.

SECTION I

- 1- a) How many degrees of freedom are associated with a Diatomic Molecule? Write down the total classical heat capacity of a diatomic molecule. 3
- b) The quantum energy levels of a rigid linear rotator is $E_j = \frac{h^2}{8\pi^2 I} j(j+1)$ where $j = 0, 1, 2, \dots$. And ' I ' is the moment of inertia about a line perpendicular to the axis and through the centre of mass. Consider only the rotational motion of a diatomic molecule:
- i) Derive the general expression of the rotational partition function and find its values at high temperatures,
- ii) Also evaluate the values of Free Energy F , energy U and specific heat. 6, 3.5

- 2- a) Distinguish between Canonical and Grand Canonical ensembles. 4
- b) The classical partition function consisting of N independent identical spinless particles, is

$$Z = \frac{1}{N! h^{3N}} \int e^{-E/\tau} d\Gamma.$$

Using the above expression of partition function, derive the following expression of entropy called as Sackur-Tetrode equation.

$$\sigma = N \log \left(\frac{V e}{N \lambda^3} \right) + \frac{3}{2} N \quad \text{where } \lambda = \frac{h}{(2\pi M \tau)^{1/2}} \text{ is the thermal de Broglie wavelength.} \quad 8.5$$

- 3- a) Describe briefly what are (i) Systems, ii) Assemblies and iii) Phase Space 3.5
- b) Derive the expression of the probability distribution $\omega(M)$ of a system of N independent particles having total magnetic moment M in the absence of magnetic field. 6
- c) If the volume of a perfect gas of N atoms is doubled, the energy being the constant, what will be the change in entropy? 3
- 4- a) Derive the following Bose-Einstein distribution: 6, 2, 4.5

$$n(\epsilon) = \frac{1}{e^{-\mu/\tau} e^{\epsilon/\tau} - 1}$$

Show that at high temperatures ($e^{\mu/\tau} \ll 1$), the above expression

- i) reduces to the classical Boltzmann distribution, and
- ii) We obtain the classical results for energy as $U \cong \frac{3}{2} N\tau$.

SECTION II

- 5- a) Derive the following Fermi-Dirac distribution and state clearly the physical reasoning used:

$$f(\epsilon) = \frac{1}{e^{(\epsilon - \epsilon_F)/\tau} + 1}$$

Sketch the graphs for the Fermi-Dirac distribution when $T = 0$ and then $T \neq 0$ and give an explanation how you obtain these sketches. 9, 3.5

- 6- a) Derive the following expression of Plank Radiation Law for the spectral distribution of radiation

inside a constant temperature enclosure: $u(\nu, T) d\lambda = \frac{8\pi h}{c^3} \frac{\nu^3}{(e^{h\nu/kT} - 1)}$

Show that the above expression can be reduced to Stefan-Boltzmann law. 6, 3

- b) What will be the Fermi temperature of an electron gas in potassium if its Fermi energy is 2.1 eV. (Boltzmann constant is $k = 1.3807 \times 10^{-23} \text{ JK}^{-1}$, 1 eV = $1.6022 \times 10^{-19} \text{ J}$) 3.5

- 7- Write notes on the following: 6, 6.5
- a) Bose-Einstein condensation
- b) Gibb's Paradox