UNIVERSITY OF GUJRAT

A WORLD CLASS UNIVERSITY

(M.A/M.Sc Part-II)

(Mathematics) Mathematical Physics

Roll No: ____________

in words: ____________

Cutting, Overwriting, Erasing, Fluid painting and use of Lead Pencil will earn no marks. Write answer of the Question No.1 and 2 on this sheet and handover it to the supervisory staff of examination within first 35 minutes.

Time Allowed: 35 Minutes ***(OBJECTIVE PART)*** ***Max. Marks: 32*** ***Sign of Supdt.*** ____________

1- a) Encircle the correct answer: *1x4*

i) $\nabla^2 \varphi = 0$ is known as ______ differential equation

a) Poisson b) Holm holtz c) Laplace d) Lagrange

ii) If K (x, t) = (x – t) form then this is called _________ Kernel

a) Symmetric b) Kronecker c) Hermitian d) None of these

iii) If $\mathcal{F}[f(x)\, e^{ax}] = F(K - ai)$ then the property is called ________ property.

a) Shifting b) Attenuation c) Convolution d) None of these

iv) To find the surface of Minimal area which is bounded by a given curve is called ________ problem.

a) Plateaus b) Dido c) Geodesic d) None of these

b) Encircle True or False: *1x8*

i) $\nabla^2 \varphi = \frac{1}{C^2}\frac{\partial^2 \varphi}{\partial t^2}$ is called three dimentional wave equation. **True / False**

ii) A set of function $\varphi_n(x)$ for n = 1, 2, 3,…, is said to be orthogonal set if

$$\int_a^b \varphi_m(x)\,\varphi_n(x)\,dx = \begin{cases} 0 & \text{if } m \neq n \\ 1 & \text{if } m = n \end{cases}$$ **True / False**

iii) $\lim\limits_{t\to 0^+} G'(x, t) - \lim\limits_{t\to 0^-} G'(x, 0) = +\frac{1}{P(t)}$ **True / False**

iv) $g(s) = f(s) + \lambda \int_a^s K(s, t)\, g(t)\, dt$ is Fredholm Integral equation of 2^{nd} kind. **True / False**

v) Laplace Transform of $\mathcal{L}[x^n] = \frac{n!}{S^{n+1}}$ **True / False**

vi) Fourior Sine Transform is defined as $f_S[f(x)] = \int_0^\infty f(x) \sin kx\, dx$ **True / False**

vii) Wave equation in plane polar form is $\frac{\partial^2 \psi}{\partial r^2} + \frac{1}{r^2}\frac{\partial \psi}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \psi}{\partial \theta^2} = \frac{1}{V^2}\frac{\partial^2 \psi}{\partial t^2}$ **True / False**

viii) The Euler Lagrange equation of the functional $I = \int_{x_1}^{x} F(x, y, y')\, dx$ is $\frac{\partial F}{\partial y'} - \frac{d}{dx}\left(\frac{\partial F}{\partial y''}\right) = 0$ **True / False**

c) Fill in the blanks meaningfully: *1x4*

i) $\nabla^2 \varphi = \frac{1}{C^2}\frac{\partial \varphi}{\partial t}$ is called three dimentional ______________________________ equation.

ii) The integral equation $f(s) = \int_a^b K(s, t)\, g(t)\, dt$ is known as ____________ equation of___________ kind.

iii) The actual path travelled by the particle is given by $A = \int_{t_1}^{t_2} (T - V)dt$ (where T for Kinetic and V for potential energies). It is called_____________________ principle.

(Continued Overleaf)

iv) Laplace Inverse formula is $\mathcal{L}^{-1}\left(\frac{A}{(S-r)^{K}}\right) =$ ____________________.

2- Give short answers of the following questions: ***2x8***

i) Classify the Homogeneous 2nd Order Linear Equation
$AU_{xx} + 2BU_{xy} + CU_{yy} + D_{ux} + EU_{y} + FU + G = 0$

ii) Give the discontinuity Condition of Green's Function.

iii) Write the General Strum Liouville Differential Equation with condition on p(x), q(x), r(x) and λ.

iv) State the term Transient Temperature Distribution.

v) Define Symmetric and Hermitian Kernal.

vi) Evaluate Fourier of $\mathcal{F}[f'(x)] =$ ____________________

vii) Define Kronecker Delta Function.

viii) Distinguish between Fredholm and Volterra Integral Equations of 1st kind.

Total Marks: 68 + 32 = 100
Pass Marks: 40%

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Roll No: ____________

(Mathematics) Mathematical Physics

Time Allowed : 2:25 hrs
Max. Marks : 68

Attempt **FOUR** Questions in all. Select **TWO** Questions from **Section A** and **TWO** Questions from **Section B.** All Questions carry equal marks.

SUBJECTIVE PART

SECTION-A

3- a) Solve $x^2 u_{xx} + xu_x + u_{yy} = 0$ by method of Separation of Variables. *9*

b) Solve the P differential equation $u_{xx} = \alpha^2 u_t$ with B. Conditions $\begin{cases} u(0, t) = 0 \quad u(\ell, t) = \rho \\ u(x, 0) = \varphi(x) \end{cases}$ *8*

4- a) Drive Heat Equation in 3-Dimensions. *8*

b) Solve the problem of a rectangular Lemma whose boundaries are kept at 0.

$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{1}{k}\frac{\partial u}{\partial t}$ with $0 \le x \le a,\ 0 \le y \le b,\ t > 0$ i.e. $\begin{cases} u(0, y, t) = 0 = u(a, y, t) \\ u(x, 0, t) = 0 = u(x, b, t) \\ u(x, y, 0) = f(x, y) \end{cases}$ *9*

5- a) Prove that the Eigenvalues of a regular (periodic) S-L System are Real. Assuming that p(x), r(x) does not vanish or changes sign in (a, b). *8*

b) In S-L problem $\frac{d^2u}{dx^2} + \lambda u = 0$ with $u'(0) = 0 = u'(\ell)$, verify the properties of S-L System. *9*

i) there are infinite number of Eigen Values with smallest but no largest. ρ

ii) the nth Eigen function has exactly (n – 1) zeroes.

iii) the Eigen functions are orthogonal and form a complete set.

SECTION-B

6- a) Solve the Integral Equation $g(s) = f(s) + \lambda \int_0^1 (1 - 3st)\, g(t)\, dt$. Find the Resolvent Kernel. *8*

b) Form the Integral equation of S.H. Oscillator Having Equation

$y'' + \omega^2 y = 0$, with condition $y(0) = 0$, $y'(0) = 1$ *9*

7- a) Using Laplace Transform Solve the Differential Equation

$y'' - y' - 2y = t^2$, with condition $y(0) = 1$, $y'(0) = 3$ *8*

b) Show that Inverse Laplace of $\mathcal{L}^{-1}\left[\frac{s^2}{(s^2 + a^2)^2}\right] = \frac{1}{2}\left(t \text{ Cosat} + \frac{1}{a} \text{ Sinat}\right)$ *9*

8- a) State and prove Fundamental Theorem of Variational Calculus.

b) Find the extremal of functional

$I(y(x), z(x)) = \int_0^{\pi/2} (y'^2 + z'^2 + 2yz)\, dx$ satisfying the Boundary Condition $\begin{cases} y(0) = 0 \quad z(0) = 0 \\ y\left(\frac{\pi}{2}\right) = 1 \quad z\left(\frac{\pi}{2}\right) = -1 \end{cases}$ *8*