UNIVERSITY OF GUJRAT

A WORLD CLASS UNIVERSITY

(Mathematics)

(M.A/M.Sc Part-I)

Algebra

Roll No: ______________

in words: ______________

Cutting, Overwriting, Erasing, Flood painting and use of Led Pencil will earn no marks. Write answer of the Question No.1 and 2 on it and handed over to the supervisory staff of examination within 35 minutes.

Time Allowed: 35 Minutes ***(OBJECTIVE PART)*** ***Max. Marks: 32*** ***Sign of Supdt.***

1- a) Encircle the correct answer: *1x4*

i) A ring having characteristics 6 is

a) Z b) Z_2 c) Z_3 d) Z_2XZ_3

ii) R/I is a field iff the ideal *I* is

a) Prime b) Maximal c) Principal d) Proper

iii) The order of quotient group $Z_6 / <3>$ is

a) 1 b) 2 c) 3 d) 4

iv) A group of order 16 cannot have a subgroup of order

a) 1 b) 2 c) 4 d) 5

b) Encircle True or False: *1x8*

i) Every division ring is simple. **True / False**

ii) In every cyclic group, every element is a generator. **True / False**

iii) All generators of Z_{20} are prime numbers. **True / False**

iv) Every ring with unity has at least two units. **True / False**

v) *nZ* has zero divisor if n is not prime. **True / False**

vi) The product of two nonunits in Z_n may be a unit. **True / False**

vii) Q is a field of quotient of $\mathbb{Z}$. **True / False**

viii) The mapping $T : \mathbb{R}^2 \to \mathbb{R}^2$ defined by T(x, y) = (2x, y + 1) is linear. **True / False**

c) Fill in the blanks meaningfully: *1x4*

i) The homomorphism from a group to itself is called____.

ii) Let H and G be two finite groups and β: H → G be an isomorphism, then ord(H) = ________________.

iii) The set of all inner outomorphism Inn (G) on a group is a____________________________of Aut(G).

iv) If a real nxn A has n distinct eigen values, then A ____.

(Continued Overleaf)

2- Give short answers of the following questions: *2x8*

i) Show that Z is a cyclic group.

ii) Show that a subring of a ring may not be an ideal.

iii) Define Prime Ideal in a ring R.

iv) Define basis of vector space and write a basis of $\mathbb{R}^3$.

v) Define a spanning set of vectors in a vector space V.

vi) If $f : Z \to Z$ is a ring homomorphism, then show that $f(1) \neq 0$.

vii) Find the characteristics polynomial and Eigenvalues for $A = \begin{bmatrix} 1 & 1 & -3 \\ 2 & 0 & 6 \\ 1 & -1 & 5 \end{bmatrix}$

(i) What is inverse of 5 in the group Z_{12}?

SUBJECTIVE PART

Total Marks: 68 + 32 = 100
Pass Marks: 40%

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Roll No: ____________

Time Allowed : 2:25 hrs
Max. Marks : 68

Attempt **FOUR** Questions in all. Select **TWO** Questions from **Section A** and **TWO** Questions from **Section B.** All Questions carry equal marks.

SECTION-A

3- a) State and prove Third Sylow's Theorem for finite groups. *9*

b) Let ϕ be a homomorphism of a group G onto another group H with Kernel K. Prove that G/K is isomorphic to K. *8*

4- a) Prove that a group G is abelian iff $G/Z(G)$ is cyclic. Where $Z(G)$ is the center of G. *9*

b) The subgroup N of G is a normal subgroup iff every left coset of N in G is a right coset of N in G. *8*

5- a) Let A be an abelian group and p be a prime number. Suppose $A_p = \{ x \, \varepsilon \, A: O(x) = p^n \text{for some } n \, \varepsilon \, Z^+\}$, then show that A_p is fully invariant. *9*

b) Prove that every group is isomorphic to a subgroup of Bijective Mappings. *8*

SECTION-B

6- a) If $f : R \rightarrow R'$ be an onto ring homomorphism, then show that R′ is isomorphic to R/ ker (f), i.e $R / \ker(f) \cong R'$. *9*

b) Let R be a commutative ring with unity. Show that ideal M is maximal in R if and only if the quotient R / M is a field. *8*

7- a) Prove that the eigenvectors of a symmetric matrix A corresponding to distinct eigenvalues are orthogonal. *9*

b) A set $\{v_1, v_2, \ldots\ldots v_n\}$ of vectors in a vector space V is linearly independent iff some v_i is the linear combination of the other vectors. *8*

8 a) Show that the vectors (3, 0, -3), (-1, 1, 2),(4, 2, -2) and (2, 1, 1) are linearly independent. *9*

b) If A and B are similar matrices, then show that they have the same determinant, the same rank, the same trace, the same characteristic polynomial and the same eigenvalues. *8*

3- a) If $f : R \rightarrow R'$ be an onto ring homomorphism, then show that R' is isomorphic to $R/\ker(f)$, i.e $R / \ker(f) \cong R'$. *9*

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SECTION-B

6- a) State and prove Sylow's Third Theorem for finite groups. *9*

b) Let ϕ be a homomorphism of a group G onto another group H with kernel K. Prove that G/K is isomorphic to K. *8*

7- a) Prove that a group G is abelian iff G/Z (G) is cyclic. Where Z(G) is the center of G. *9*

b) The subgroup N of G is a normal subgroup iff every left coset of N in G is a right coset of N in G. *8*

8- a) If H and K are finite subgroups of G of orders o(H) and o(K) respectively, then $o(HK) = \dfrac{o(H)o(K)}{o(H\cap K)}$ *9*

b) Prove that every group is isomorphic to a subgroup of bijective mapping. *8*