UNIVERSITY OF GUJRAT

A WORLD CLASS UNIVERSITY

(M.A/M.Sc Part-I) (Mathematics)

Complex Analysis & Differential Geometry

Roll No: ____________

in words: ____________

Cutting, Overwriting, Erasing, Fluid painting and use of Lead Pencil will earn no marks. Write answer of the Question No.1 and 2 on this sheet and handover it to the supervisory staff of examination within first 35 minutes.

Time Allowed: 35 Minutes ***(OBJECTIVE PART)*** ***Max. Marks: 32*** ***Sign of Supdt.***

1- a) Encircle the correct answer: ***1x4***

i) Radius of convergence of power series $\sum\left(1+\frac{1}{n}\right)^{n^n} Z^n$ is

a) e b) $\frac{1}{e}$ c) Zero d) None of these

ii) If a function of $f(z)$ involves $\overline{z}$ then without verifying C.R equations we can say function is

a) Analytic b) Harmonic c) Not analytic d) None of these

iii) $\underline{R} = \underline{r} + u\underline{n}$ is equation of

a) Osculating Plane b) Principal Normal

c) Rectifying Plane d) None of these

iv) $f(z) = \frac{5}{z^2} - \frac{6}{z^5} + \frac{7}{z^8}$ has a pole of order

a) 2 b) 5 c) 7 d) None of these

b) Indicate True or False: ***1x8***

i) A curve which is simple as well as closed is called Jordan Curve. …………………. **True / False**

ii) Green's Theorem is used in Cauchy's fundamental theorem. ………………………. **True / False**

iii) Cos h z is periodic function of period π. …………………………. **True / False**

iv) $|Z_1 - Z_2| \geq \big||Z_1| - |Z_2|\big|$ ……………………. **True / False**

v) A linear transformation is conformal mapping. ……………………………………. **True / False**

vi) The function $f(z) = \frac{1}{z-1}$ has an isolated singularity at z = 1 ……………… **True / False**

vii) $\underline{n}' = K\,\underline{n}$ ……………………………………………….. **True / False**

viii) In first order Magnitude $E = r_1^2$ ………………………...……………….………… **True / False**

c) Fill in the blanks meaningfully: ***1x4***

i) A transformation AZ + B = W where A and B are complex constants A ≠ o is called. ____________.

ii) $\underline{t} \times \underline{b}$ = ____________________________; $\underline{b} \cdot \underline{t}$ = ______________________________________.

iii) For Laplace equation in Polar form $\frac{\partial^2 u}{\partial r^2}$ = ____.

iv) Log Z = ____ for $-\pi < \theta < \pi$

(Continued Overleaf)

2- Answer the of following questions: *2x8*

i) Define Conformal and Isogonal Mappings.

ii) Define Circular Helix.

iii) State Mittag Leffer's Theorem.

iv) Find $T^{-1}(W)$ for the transformation $T(Z) = \dfrac{Z - 2}{Z + 5}$

v) Prove that $r'' \cdot r''' = K K'$

vi) Write the conditions that a surface is developable.

vii) State Cauchy Residue Theorem.

viii) Define Uniform Continuity.

SUBJECTIVE PART

Total Marks = 68 + 32=100
Pass Marks: 40%

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(M.A/M.Sc Part-I) (Mathematics)

Complex Analysis & Differential Geometry

Roll No: ____________

Time Allowed : 2:25 hrs
Max. Marks : 68

Attempt **FOUR** Questions in all. Select **TWO** Questions from **Section A** and **TWO** Questions from **Section B.** All Questions carry equal marks.

SECTION-A

3- a) If $Z = \dfrac{(1+i)+(3+2i)t}{1+i\,t}$ then prove that the locus of Z is a circle. Find radious and centre of circle, also calculate the minimum and maximum distance of Z from origin. *9*

b) If W_1, W_2, W_3 and W_4 be the images of the four distincit points Z_1, Z_2, Z_3 and Z_4 in the z plane under a bilinear transformation. $W = T(Z) = \dfrac{az+b}{cz+d}$ where $ad - bc \neq 0$ *8*

4- a) Prove that $\displaystyle\int_{-\infty}^{\infty} \frac{x^2\,dy}{(x^2+a^2)^3} = \frac{\pi}{8a^3}$ Provided that R(a) is positive. *9*

b) $\displaystyle\int_{0}^{\infty} \frac{\cos mx}{a^2+x^2}\,dx$ *8*

5- a) Find the edge of regression of the envelope of family of plane.

$x \sin\theta - y\cos\theta + z = a\theta$ θ being parameter *9*

b) If $u = x^3 - 3xy^2 + 3x^2 - 3y^2 + 1$ then u (x, y) is harmonic, construct the original function f (z) *8*

SECTION-B

6- a) Evaluate $\displaystyle\int_{0}^{3+i} z^2\,dz$ along C where C is

i) $\overline{OA}$ ii) $\overline{AB}$ iii) $\overline{OB}$ iv) OABO *9*

b) If the tangent and binormal at a point of a curve make angles θ, ϕ respectively with a fixed direction show that $\dfrac{\sin\theta}{\sin\phi}\,\dfrac{d\theta}{d\phi} = \dfrac{-K}{\tau}$. *8*

7- a) Prove that $[\underline{t}'\ \ \underline{t}''\ \ \underline{t}'''] = [\underline{r}''\ \ \underline{r}'''\ \ \underline{r}''''] = K^3(K\tau' - K'\tau) = K^5\dfrac{d}{ds}\left(\dfrac{\tau}{K}\right)$ *9*

b) State and prove Cauchy's Integral Formula. *8*

8- a) Prove that every bilinear transformation with two finite fixed points α, β can be put in the normal form *9*

$$\frac{W-\alpha}{W-\beta} = K\left(\frac{z-\alpha}{z-\beta}\right)$$

b) Calculate the fundamental magnitudes for the heliciod given by

$x = u\cos\phi$, $y = u\sin\phi$; $z = c\,\phi$ with u, ϕ as parameters. *8*