UNIVERSITY OF GUJRAT

A WORLD CLASS UNIVERSITY

(Mathematics)

(M.A/M.Sc Part-II)
Number Theory

Roll No: ____________

in words: ____________

Cutting, Overwriting, Erasing, Fluid painting and use of Lead Pencil will earn no marks. Write answer of the Question No.1 and 2 on this sheet and handover it to the supervisory staff of examination within first 35 minutes.

Time Allowed: 35 Minutes ***(OBJECTIVE PART)*** ***Max. Marks: 32*** ***Sign of Supdt.***

1- a) Encircle the correct answer: ***1x4***

i) $d(3^5)$ is

a) 5 b) 3 c) 6 d) 15

ii) σ (10) is

a) 12 b) 14 c) 16 d) 10

iii) φ (10) is

a) 4 b) 6 c) 8 d) 10

iv) Solution of Congurence $2x \equiv 1 \pmod 5$ is

a) $x \equiv 1 \pmod 5$ b) $x \equiv 2 \pmod 5$ c) $x \equiv 3 \pmod 5$ d) $x \equiv 4 \pmod 5$

b) Indicate True or False: ***1x8***

i) Linear Diophantine Equation $18x + 24y = 3$ has solution. **TRUE / FALSE**

ii) The set {1, 2, 3, ….. 12} is RRS mod 13 **TRUE / FALSE**

iii) $\varphi(n) \neq 1$ if $n = 1$ **TRUE / FALSE**

iv) 3 is primitive root mod 7 **TRUE / FALSE**

v) The exponent of 2 modulo 8 is not defined. **TRUE / FALSE**

vi) 4 is quadratic residue mod 5. **TRUE / FALSE**

vii) If $x^2 \equiv 3 \pmod{13}$ has a solution $\Rightarrow \left(\frac{3}{12}\right) = -1$ **TRUE / FALSE**

viii) The product of primitive polynomials is not primitive. **TRUE / FALSE**

c) Fill in the blanks meaningfully: ***1x4***

i) If (a, b) = d, then [a, b] ______________________________.

ii) Φ (þ) = ______________________________ where þ is prime.

iii) If $a_1, a_2, a_3, \ldots\ldots A_k$ form RRS (mod m) then K = ______________________________.

iv) μ(20) = ______________________________.

(Continued Overleaf)

2- Give short answers the following questions: ***2x8***

i) Evaluate (256, 1166) = ?

ii) Solve the convergence $5x \equiv 2 \pmod{26}$

iii) Show 4 is not Primitive Root mod 7.

iv) Evaluate $\left(\frac{2}{31}\right)$

v) If $n = þ^{\alpha}$. Show $d(n) = (\alpha + 1)$ where þ is Prime

vi) If $n = þ^{\alpha}$ where þ is prime then show $\sum_{d|n} \varphi(d) = n$

vii) Define Fesmats number with example.

viii) Define Minimal Polynomial.

Total Marks: 68 + 32 = 100
Pass Marks = 40%

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Roll No: ______________

Time Allowed : 2:25 hrs
Max. Marks : 68

Attempt **FOUR** Questions in all. Select **TWO** Questions from **Section A** and **TWO** Questions from **Section B.** All Questions carry equal marks.

SUBJECTIVE PART

Section-A

3- a) Let a and b can be any two positive integers then (a, b) [a, b] = ab *9*

b) Let m be a positive integer, a and b any integers, the linear congruence $ax \equiv b \pmod{m}$. Has solution if and only if $d \mid b$, d = (a, m). In case $d \mid b$, the given congruence has d mutually incongruent solutions mod m. *8*

4- a) Let þ be an odd prime, prove that *9*

i) $1^{þ} + 2^{þ} + 3^{þ} + \ldots\ldots (þ - 1)^{þ} \equiv 0 \pmod{þ}$

ii) $1^{þ-1} + 2^{p-1} + \ldots\ldots\ldots (þ - 1)^{þ-1} \equiv -1 \pmod{þ}$

b) State and prove Wilson's Theorem. *8*

5- a) If (b, c) = 1 and b c is a perfect square, show that both b and c are perfect squares. *9*

b) Let þ be a prime and n is positive integer then exponent e such that $þ^{e} \mid n!$ is at most $\sum_{i=1}^{\alpha} \frac{n}{þ^{i}}$ *8*

Section-B

6- a) Define Fermals Number and show that two distinct fermats numbers are relatively prime. *9*

b) Define primitive root and find all primitive roots mod 7. *8*

7- a) Show that there Enist Transcendental Number. *9*

b) A multiple algebraic extension of F is a simple algebraic extension. *8*

8- a) State and prove Theorem relating with Primitive Polynomials. *9*

b) State and prove Lagrange's Theorem on Polynomial Congruence. *8*