UNIVERSITY OF GUJRAT

A WORLD CLASS UNIVERSITY

(M.A/M.Sc Part-I)
(Mathematics)
Topology and Functional Analysis

Roll No: ______________

in words: ______________

Cutting, Overwriting, Erasing, Fluid painting and use of Lead Pencil will earn no marks. Write answer of the Question No.1 and 2 on this sheet and handover it to the supervisory staff of examination within first 35 minutes.

Time Allowed: 35 Minutes ***(OBJECTIVE PART)*** ***Max. Marks: 32*** ***Sign of Supdt.***

1- a) Tick or Encircle the correct answer: ***1x4***

i) $A = [a, b] \Rightarrow \mathring{A} =$

a)]a, b[b)]a, b] c) [a, b] d) [a, b[

ii) Every finite T_1 – space is

a) Indiscrete b) Discrete c) Uncountable d) ϕ

iii) Let d_o be a discrete metric on R, then $d_o(7, 9) =$

a) 3 b) 2 c) 1 d) 0

iv) If V is an inner product space, then for all $x \in V$,

a) $<0, x> = 0$ b) $<0, x> \neq 0$ c) $<0, x> = 1$ d) $<0, x> = -1$

b) Indicate True or False: ***1x8***

i) Every subset of a discrete topological space is open. **True / False**

ii) $F_r(A) = \overline{A} \cup \overline{A}^{c}$ **True / False**

iii) Intersection of any number of open sets is open. **True / False**

iv) The continuous image of compact space is compact. **True / False**

v) In metric space every convergent sequence is a Cauchy sequence. **True / False**

vi) A closed ball in a metric space is a closed set. **True / False**

vii) If *f* is homeomorphism, then *f* is not bijective. **True / False**

viii) A normed space X is said to be reflexive if there is an isometric isomporhism between X and X**. **True / False**

c) Fill in the blanks meaningfully: ***1x4***

i) A sub-collection of an open cover which is itself an open cover is called a ________________________.

ii) Every completely regular space is ____.

iii) The limit of a convergent sequence in a normed space is ____.

iv) Every finite dimensional subspace Y of a normed space X is _______________________________ in X.

(Continued Overleaf)

2- Give short answers of the following questions: *2x8*

i) Define the subspace of a topological space.

ii) If A and B are two subsets of a topological space X, then show that $\overline{A \cap B} \subseteq \overline{A} \cap \overline{B}$

iii) Let $X = \{a, b, c, d\}$ with topology $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$, then find all possible neighbourhoods of *a*.

iv) Define a Normal Space.

v) Define a Discrete Metric Space.

vi) If A and B are two subsets of a metric space, then show that $A \subseteq B \Rightarrow A^{o} \subseteq B^{o}$.

vii) Define Quotient Space.

viii) In an inner product space X, show that $\langle x, z\rangle = \langle y, z\rangle \Rightarrow x = y, \forall z \in X$.

SUBJECTIVE PART

Total Marks = 68 + 32 = 100

Pass Marks = 40%

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Time Allowed : 2:25 hrs

Max. Marks : 68

Attempt any **FOUR** Questions in all. All Questions carry equal marks.

3- a) Show that a function f from one topology space X into another topolofical space Y is continuous if and only if for every closed set C in Y, f^{-1} (C) is closed in X.

b) Show that a collection β of subsets of a topological space (X, τ) is a base for τ if and only if

i) every point of X is in some $B \in \beta$.

ii) For $B_1, B_2 \in \beta$ and $x \in B_1 \cap B_2$, there is a $B \in \beta$ such that $x \in B \subseteq B_1 \cap B_2$.

4- a) Show that a closed subspace of Lindelof space is Lindelof.

b) A topological space X is a T_1-space iff each one point set in X is closed.

5- a) Show that a first countable space is Hausdorff space if and only if every convergent sequence has a unique limit.

b) Show that a subspace of the real line R is connected if and only if it is an interval.

6- a) Prove that every convergent sequence in a metric space is a Cauchy sequence.

b) Prove that a subspace Y of a Banach space X is Banach if and only if Y is closed in X.

7- a) Let Y be a closed subspace of a Hilbert space H, then show that $H = Y \oplus Y^{\perp}$.

b) Let $T : X \rightarrow Y$ be a linear operator from a normed space X into a normed space Y, then show that T is continuous iff T is bounded. *9*

8- a) Prove that every compact subspace of a Hoxsdorff space is closed.

b) For any two elements x, y of an inner product space V, show that $\|x + y\|^2 + \|x - y\|^2 = 2\|x\|^2 + 2\|y\|^2$