UNIVERSITY OF GUJRAT

A WORLD CLASS UNIVERSITY

(Mathematics)

(M.A/M.Sc Part-II)

Rings and Modules

Roll No: ______________

in words: ______________

Cutting, Overwriting, Erasing, Fluid painting and use of Lead Pencil will earn no marks. Write answer of the Question No.1 and 2 on this sheet and handover it to the supervisory staff of examination within first 35 minutes.

Time Allowed: 35 Minutes ***(OBJECTIVE PART)*** ***Max. Marks: 32*** ***Sign of Supdt.***

1- a) Encircle the correct answer: *1x4*

i) Let $\alpha : R \to S$ be a ring homomorphism. Then ker α is

a) equal to {0} b) an ideal of S

c) an ideal of R d) None of these

ii) If the {0} ideal of a commutative ring R with I is prime, then R is:

a) Integral domain b) Field c) Principal ideal domain d) None of these

iii) Let $f : M \to N$ be an R- homomorphism and A be a submodule of M then f (A) is a submodule of

a) M b) N c) M + N d) None of these

iv) Let F be a field. Then [F : F] is equal to:

a) Zero b) 1 c) 2 d) None of these

b) Encircle True or False: *1x8*

i)	Every finite integral domain is a field.	**TRUE / FALSE**
ii)	If F is a field, then F[x] is a field.	**TRUE / FALSE**
iii)	x^2 - 2 is irreducible over Q.	**TRUE / FALSE**
iv)	$M_2(R)$ has no divisors of 0, where R is the set of real numbers.	**TRUE / FALSE**
v)	Every module is a homomorphic image of some free module.	**TRUE / FALSE**
vi)	Every abelian group under addition is a $\mathbb{Z}$-module.	**TRUE / FALSE**
vii)	Every ring with identity is a free module over itself.	**TRUE / FALSE**
viii)	Every vector space over a field F is a free module.	**TRUE / FALSE**

c) Fill in the blanks meaningfully: *1x4*

i) If $f : M \to N$ is a module homomorphism, then $M/_{\text{Ker } f} \cong$ ________________________________.

ii) An R-module with a linearly independent, spanning set is called________________________________.

iii) An integral domain in which each ideal is principal is called________________________________.

iv) Two element a, b of an integral domain R are called associates if ________________________________.

(Continued Overleaf)

2- Give short answers of the following questions: *2x8*

i) Define Associate Elements.

ii) Define Algebraic Element.

iii) Define Unique Factorization Domain.

iv) What is Algebraic Extension of a Field?

v) Define quotient module of an R-module.

vi) Define Torsion Module.

vii) Define Cyclic Module and give an example of a cyclic module.

viii) Let $f : M \to N$ be a module homomorphism show that Ker f is a submodule of M.

Total Marks: 68 + 32 = 100
Pass Marks = 40%

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(M.A/M.Sc Part-II) **Roll No:** ____________

(Mathematics) Rings and Modules

Time Allowed : 2:25 hrs
Max. Marks : 68

Attempt **FOUR** Questions in all. Select **TWO** Questions from **Section A** and **TWO** Questions from **Section B.** All Questions carry equal marks.

SUBJECTIVE PART

SECTION A

3- a) Let A, B are submodules of an R-module M. Show that $(A + B)/_B$ is isomorphic to $A/_{(A \cap B)}$. *9*

b) Let A, B, C are submodule of an R-module M such that $A \subseteq B \cup C$. Show that either $A \subseteq B$ or $A \subseteq C$. *8*

4- a) Let A, B are submodules of an R-module M. Show that M is a direct sum of A and B if and only if each element m of M can be uniquely written as $m = a + b$ where $a \in A$ and $b \in B$. *9*

b) Show that every module is a homomorphic image of some free module. *8*

5- a) Show that two cyclic R-modules are isomorphic if and only if they have same order ideal. *9*

b) Show that every vector space V over a field F considered as an F-module is torsion free. *8*

SECTION B

6- a) Let E be a simple extension F (α) of a field F, and Let α be algebraic over F. Let the degree of irr (α, F) be $n \geq 1$. Then every element β of E = F (α) can be uniquely expressed in the form $\beta = b_0 + b_1\alpha + \ldots\ldots\ldots + b_{n-1}\alpha^{n-1}$ where the b_i are in F. *9*

b) If E is a finite extension field of a field F, and K is a finite extension field of E, then K is a finite extension of F, and $[K : F] = [K : E][E : F]$ *8*

7- a) Prove that a polynomial of degree n over a field can have at most n roots in any extension of the field. *9*

b) Show that every principal ideal domain is a unique factorization domain. *8*

8- a) Show that $\mathbb{Z}[r]$ is a Euclidean domain. *9*

b) Show that every Euclidean domain is a principal ideal domain. *8*