

Cutting, Overwriting, Erasing, Fluid painting and use of Lead Pencil will earn no marks.  
 Write answer of the Question No.1 and 2 on this sheet and handover it to the supervisory staff of examination within first 35 minutes.

**Time Allowed: 35 Minutes**

**(OBJECTIVE PART)**

**Max. Marks: 32**

**Sign of  
Supdt.**

**1- a) Encircle the correct answer:**

1x4

\_\_\_\_\_

- i) An interval on  $\mathbb{R}$  is
- a) Finite                      b) Countable                      c) Uncountable                      d) None of These
- ii) If  $E$  is Lebesgue measureable set then  $E^c$  is
- a) Non-measurable                      b) Measurable  
 c) Countable                      d) None of these
- iii) The cardinal number of  $\mathbb{R}$  is
- a) 5                      b)  $N_0$                       c)  $\mathbb{C}$                       d) None of these
- iv) Lebesgue Outer measure of Cantor Set is
- a)  $\infty$                       b) 0                      c) 1                      d) None of these

**b) Encircle True or False:**

1x8

**True / False**

**True / False**

**True / False**

**True / False**

**True / False**

**True / False**

**True / False**

**True / False**

**c) Fill in the blanks meaningfully:**

1x4

- i) Multiplication in ordinals is not \_\_\_\_\_.
- ii) Last element in a TOSET always \_\_\_\_\_.
- iii) Lebesgue measure of an interval is its \_\_\_\_\_.
- iv) Lebesgue Outer measure of cantor set  $\equiv$  \_\_\_\_\_.

(Continued Overleaf)

2- Give short answers of the following questions:

2x8

i) First element of a POSET is unique. \_\_\_\_\_

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ii) Set of all integers  $\mathbb{Z}$  is not a Woset. Explain.

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iii) Write the statement of Fatou's Lemma. \_\_\_\_\_

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iv) Lebesgue Outer Measure of singleton set is zero. \_\_\_\_\_

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v) Define  $G_\delta$  Set. \_\_\_\_\_

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vi) Define Legendre's differential equation of degree  $n$ . \_\_\_\_\_

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vii) Express Hypergeometric Function  $F(a, b; c; x)$  as an Infinite Series.

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viii) Write the generating function for Bessel's function.

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# (Mathematics) Advanced Analysis

Attempt **FOUR** Questions in all. Select **TWO** Questions from **Section A** and **TWO** Questions from **Section B**. All Questions carry equal marks.

## SUBJECTIVE PART

### SECTION-A

- 3- a) Prove that the interval  $[0, 1]$  is not denumerable. 9
- b) Prove that the set for any set  $A$ ,  $A \propto 2^A$  and hence  $\#(A) < \#(2^A)$  8
- 4- a) Prove that two different initial segments of a Woset cannot be similar 9
- b) For positive integer  $n$  show that  $J_{-n}(x) = (-1)^n J_n(x)$ . 8
- 5- a) Show that  $P_0(x) = 1, P_1(x) = x, P_2(x) = \frac{3x^2 - 1}{2}, P_3(x) = \frac{1}{2}(5x^3 - 3x)$  9
- b) If  $f(x) = \sum_{n=0}^{\infty} b_n P_n(x)$  then show that  $b_n = \frac{2n+1}{2} \int_{-1}^1 P_n(x) f(x) dx$  8

### SECTION-B

- 6- a) Let  $A$  be any set of real numbers and  $\{E_i\}$  sequence of pair wise disjoint measurable sets then
- $$m^*\left(A \cap \left(\bigcup_{i=1}^{\infty} E_i\right)\right) = \sum_{i=1}^{\infty} m^*(A \cap E_i)$$
- 12
- b) Prove that the Lebesgue Outer Measure of a finite closed interval is its length. 5
- 7- a) Show that the Cantor Set  $C$  has Lebesgue measure zero. 9
- b) Prove that Non-Lebesgue Measurable Sets exists. 8
- 8- a) Let  $f$  be any real valued function defined on a measurable domain  $D$  and  $G$  an open set in  $\mathbb{R}$ .  
 Then  $f$  is measurable. If and only if  $f^{-1}(G)$  is measurable. 9
- b) State and prove Bounded Convergence Theorem. 8