UNIVERSITY OF GUJRAT

A WORLD CLASS UNIVERSITY

(Mathematics)

(M.A/M.Sc Part-I)

Mechanics

Roll No: ____________

in words: ____________

Cutting, Overwriting, Erasing, Fluid painting and use of Lead Pencil will earn no marks. Write answer of the Question No.1 and 2 on this sheet and handover it to the supervisory staff of examination within first 35 minutes.

Time Allowed: 35 Minutes *(OBJECTIVE PART)* *Max. Marks: 32*

Sign of Supdt. ____________

1- a) Encircle the correct answer: *1x4*

i) $\nabla.\hat{r} =$

a) 3 b) 1 c) $\underline{r}/r$ d) None of these

ii) $\in_{ijj}$ is a tensor of rank

a) 3 b) 2 c) 1 d) None of these

iii) If A_i , B_{jk} and $C_{\ell mn}$ are tensors of rank 1, 2, and 3 respectively then $A_i\ B_{\ell i}\ C_{\ell mn}$ is a tensor of rank.

a) 6 b) 4 c) 3 d) 2

iv) M.I of a circular disc of mass 'm' and radious 'b' about a diameter is

a) $\frac{mb^2}{4}$ b) mb^2 c) $\frac{mb^2}{2}$ d) None of these

b) Encircle True or False: *1x8*

i) For an irrational vector field $\underline{F}$, $\nabla \times \underline{F} \neq 0$ **True / False**

ii) $\nabla \times (\nabla \times \underline{A}) = \nabla (\nabla . \underline{A}) + \nabla^2 \underline{A}$ **True / False**

iii) In cylindrical polar coordinates, the volume element is $dv = d\rho\ d\phi\ dz$ **True / False**

iv) $\delta_{ij}\ \delta_{ij} = 3$ **True / False**

v) If A_{ij} is skew symmetric tensor, then $A_{ij} = 0$ when $i = j$ **True / False**

vi) In general rigid body motion, the linear displacements of all the particles on axis of rotation are same. **True / False**

vii) M.I of a particle about a line is independent of its distance from that line. **True / False**

viii) Principal axes of inertia are the axes relative to which products of inertias are non zero. **True / False**

c) Fill in the blanks meaningfully: *1x4*

i) $\nabla (f(r)) =$ ____.

ii) In case of orthogonal curvilinear coordinates $\nabla u_1 \times \nabla u_2 . \nabla u_3 =$ ____________________.

iii) $\delta_{ij}\ \delta_{jk}\ \delta_{ki} =$ ____.

iv) The number of degree of freedom of a particle moving along a plane curve is____________.

(Continued Overleaf)

2- Give short answers of the following questions: ***2x8***

i) Prove that the vector field $\underline{A} = \frac{\underline{r}}{r}$ is irrotational.

ii) From the relation $\underline{v} = \underline{w} \times \underline{r}$ for the circular motion of a particle if w is constant show that $\nabla \times \underline{v} = 2\underline{w}$

iii) If $\underline{r}$ is a positio vector, then prove that $\iint_S \underline{r} \cdot d\underline{s} = 3v$, where v is the volume enclosed by the surface S.

iv) Describe coordinate surfaces in rectangular Cartesian Coordinate System.

v) For the 2^{nd} order tensor A_{ij}, show that A_{ii} is an invariant.

vi) Show that under an orthogonal transformation, a 2^{nd} order symmetric tensor remains a 2^{nd} order symmetric tensor.

vii) Find the number of degree of freedom for a Particle moving along a space curve.

viii) State Principle of Angular Momentum.

SUBJECTIVE PART

Total Marks: 68+32=100
Pass Marks = 40%

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Mechanics

Roll No: ______________

Time Allowed : 2:25 hrs
Max. Marks : 68

Attempt **FOUR** Questions in all. Selecting at least one question from each section. All Questions carry equal marks.

SECTION-A

3- a) Find the directional derivative of $\phi = 4e^{2x-y+z}$ at the point (1, 1, -1) in a direction towards the point (-3, 5, 6) *9*

b) Evaluate $\int_S \int \underline{A}.\, \hat{n}\, ds$ where $\underline{A} = z\hat{i} + x\hat{j} - 3y^2 z\hat{k}$ and s is the surface of the cylinder $x^2 + y^2 = 16$, included in the 1st octant between Z = 0 and Z = 5 *8*

4- a) Prove that every second (rank) order tensor A_{ij} can be written as the sum of a Traceless and an isotropic tensor. *8*

b) Express the velocity and acceleration of a particle in cylindrical polar coordinates. *9*

SECTION-B

5- a) Define contraction of a tensor. Prove that a single contraction of a tensor of rank 'r', yields a tensor of rank 'r – 2'. *8*

b) Prove that if $A_{(ijk)} B_{ij}$ is a vector, where B_{ij} is an arbitrary tensor of rank 2, then $A_{(ijk)}$ is also a 3rd rank tensor. *9*

6- a) Prove that if A_{ijk} is a tensor of rank 3 then $\dfrac{\partial^2 A_{ijk}}{\partial x_p\, \partial x_q}$ is a tensor of rank 5. *8*

b) Prove the identity by using tensor method $\underline{A} \times (\underline{B} \times \underline{C}) = (\underline{A} . \underline{C})\, \underline{B} - (\underline{A} . \underline{B})\, \underline{C}$ *9*

SECTION-C

7- a) A rigid body with one point fixed moves freely in space. Show that its any displacement is equivalent to a rotation about some axis, passing through that fixed point. *8*

b) Find moment of Inertia of a triangular plate of mass 'm' about one of its sides. *9*

8- a) A square of side 'a' has particles of masses m, 2m, 3m and 4m at its vertices. Find the principal moments of inertia at the centre of the square. *9*

b) Derive Euler's dynamical equations for the motion of a rigid body fixed at one point. *8*