UNIVERSITY OF GUJRAT

A WORLD CLASS UNIVERSITY

(Mathematics)

(M.A/M.Sc Part-I)

Real Analysis

Roll No: _____

in words: _____

Cutting, Overwriting, Erasing, Fluid painting and use of Lead Pencil will earn no marks. Write answer of the Question No.1 and 2 on this sheet and handover it to the supervisory staff of examination within first 35 minutes.

Time Allowed: 35 Minutes ***(OBJECTIVE PART)*** ***Max. Marks: 32***

Sign of Supdt.

_____

1- a) Encircle the correct answer: *1x4*

i) If A $\{x : 0 < x \leq 1\}$ then set A is

a) Bounded b) Bounded above

c) Bounded below d) None of these

ii) An infinite series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ diverges if

a) $p < 1$ b) $p \leq 1$ c) $p > 1$ d) $p \geq 1$

iii) Every closed subset of a compact space is _____

a) Compact b) Not compact c) Open d) Connected

iv) Every continues function on [a, b] _____ of bounded variation.

a) is always b) is never c) may not be d) is occasionally

b) Encircle True or False: *1x8*

i) ~~$\forall$~~ a, b, c $\in$ IR. If a > b and c < 0 then ac < bc. **True / False**

ii) Every non empty set of real numbers that has an upper bounded has a supremum in IR. **True / False**

iii) Let $f = I \rightarrow IR$ be a continuous function on a closed bounded interval then f may be the uniformly continuous on I. **True / False**

iv) An infinite geometric series $\sum_{n=0}^{\infty} \gamma^n$ converges when $\gamma < 1$ **True / False**

v) F is differentiable on I = [a, b] and number K is between $f(a)$ and $f'(b)$ then there is at least one point c on]a, b[such that $f'(c) = K$ **True / False**

vi) Suppose $f \in R[a, b]$ then its absolute value $|f|$ is in R [a, b] **True / False**

vii) A function of bounded variation can be expressed as a difference of two monotone increasing functions. **True / False**

viii) f and g are positive such that $f(x) \leq g(x)$ then $\int g(x)\,dx$ converges if $\int f(x)\,dx$ converges. **True / False**

c) Fill in the blanks meaningfully: *1x4*

i) f is said to be of bounded variation on [a, b] iff V (f, a, b) is _____.

ii) A real valued function f is said to be continuous at $c \in]a, b[\subseteq IR$ if $\lim_{x \to c} f(x)$_____.

iii) An alternating series $\sum_{n=1}^{\infty} (-1)^{n-1} u_n$ converges if _____.

iv) If ϕ is bounded and monotonic on $[a, \infty[$ and $\int_a^{\infty} f(x)\,dx$ is convergent at ∞ then $\int_a^{\infty} f(x) \cdot \phi(x)dx$ is

_____.

(Continued Overleaf)

2- Give short answers of the following questions: *2x8*

i) Define Inner Product. _____

_____

_____

ii) What is meant by a real sequence?

_____

_____

_____

iii) Give the basic concept of the uniform continuity. _____

_____

_____

_____

iv) State comparison integral test for convergence._____

_____

_____

_____

v) What is Cauchy's criterion for uniform convergence of sequence of functions?

_____

_____

_____

vi) Define Subsequential Limit. _____

_____

_____

_____

vii) Write down the beta function and give conditions for its convergence.

_____

_____

_____

viii) Give a brief note on the total variation of function *f* on [a, b]

_____

_____

_____

Total Marks: 68 + 32 = 100
Pass Marks: 40%

UNIVERSITY OF GUJRAT
A WORLD CLASS UNIVERSITY

(Mathematics)

(M.A/M.Sc Part-I)
Real Analysis

Roll No: _____

Time Allowed : 2:25 hrs
Max. Marks : 68

Attempt **FOUR** Questions in all. Select **TWO** Questions from **Section A** and **TWO** Questions from **Section B.** All Questions carry equal marks.

SUBJECTIVE PART

SECTION-A

3- a) Prove that Sup(-A) = -Inf A where A is the bounded subset of $\mathbb{R}$. *9*

b) Show that a Seq $\{u_n\}$ defined by $u_n = a + \frac{1}{L^1} + \frac{1}{L^2} + \ldots\ldots + \frac{1}{L^n}$ is convergent and $2 \leq \lim_{n\to\infty} u_n \leq 3$. *8*

4- a) Investigate the behaviour of convergence for $\sum_{2}^{\infty} \frac{1}{n(\log n)^p}$ $\forall$ $n \in N$ and $p \in \mathbb{R}$. *9*

b) Prove that the function f defined by $f(x) = \frac{1}{x}$ on an interval]0,1] is continuous but not uniformly continuous on that interval. *8*

5- a) A derivable function y = f(x) at any point is necessarily continuous at that point but not conversely. *9*

b) Give $f(x, y) = \begin{cases} \frac{x^2 y}{x^4 + y^2} & (x,y) \neq 0 \\ 0 & (x,y) = 0 \end{cases}$

Prove that f is discontinuous at (0, 0). Do $f_x(0, 0)$ and $f_y(0, 0)$ exist in any direction? *8*

SECTION-B

6- a) Show that $f(x) = \begin{cases} x \sin\frac{\pi}{x} & ; \quad 0 < x \leq 1 \\ 0 & ; \quad x = 0 \end{cases}$ is not of bounded variation. *9*

b) Show that $\lim_{n\to\infty} \frac{n^2}{(n+0)^3} + \frac{n^2}{(n+1)^3} + \ldots\ldots\ldots\ldots + \frac{n^2}{(n+n)^3} = \frac{3}{8}$ *8*

7- a) Test the sequence $\langle f_n \rangle$ for uniform convergence where $f_n(x) = \frac{nx}{1 + n^2 x^2}$ $\forall$ $x \in \mathbb{R}$. *9*

b) Show that the series $1 - x + x^2 - x^3 + \ldots\ldots\ldots\ldots$ is not uniformly convergent on [0, 1] however it admits term by term integration. *8*

8- Investigate convergence of improper integral *9,8*

a) $\int_a^b \frac{dx}{(x-a)^n}$ b) $\int_1^\infty \frac{dx}{x^{1/3}(1+x)}$