UNIVERSITY OF GUJRAT

A WORLD CLASS UNIVERSITY

(B.A/B.Sc. Part-I) (New & Old Syllabus) **Roll No:** ______

Mathematics B-Course (Paper-II)

Time Allowed : 3 hrs
Max. Marks : 50
Pass Marks : 33%

Attempt **Five** Questions in all. Select **Two** Questions from **Section-A** and **Three** from **Section-B.**

Section-A

1- a) Prove that $[\underline{a} + \underline{b}, \underline{b} + \underline{c}, \underline{c} + \underline{a}] = 2[\underline{a}\ \underline{b}\ \underline{c}]$. 5

b) Show that $[\underline{a} \times \underline{b}, \underline{b} \times \underline{c}, \underline{c} \times \underline{a}] = [\underline{a}\ \underline{b}\ \underline{c}]^2$. 5

2- a) If $\underline{\mathbf{r}} = (\text{Cos } n\, t)\, \underline{i} + (\text{Sin } n\, t)\, \underline{j}$ where n is a constant show that $\underline{\mathbf{r}} \times \frac{d\underline{\mathbf{r}}}{dt} = n\underline{k}$. 5

b) If $\underline{v}$ is a vector function, solve the equation $\frac{d^2\underline{v}}{dt^2} = at + b$, $\underline{v} = a\frac{t^3}{6} + b\frac{t^2}{2}$. 5

3- a) Find the scalar function φ such that $\nabla\varphi = x\underline{i} + 2y\underline{j} + z\underline{k}$. 5

b) If $\underline{\mathbf{r}} = x\underline{i} + y\underline{j} + z\underline{k}$, show that $\nabla(\mathbf{r}^n) = n\, \mathbf{r}^{n-1}\, \hat{\underline{\mathbf{r}}}$ 5

Section-B

4- a) State and prove (λ, μ) theorem. 5

b) The greatest resultant that two forces can have is of magnitude P and the least is of magnitude Q. Show that, when they act at an angle α, their resultant is of magnitude $\sqrt{P^2 \text{Cos}^2 \frac{\alpha}{2} + Q^2 \text{Sin}^2 \frac{\alpha}{2}}$. 5

5- a) Find the least force which will set into motion a particle at rest on a rough horizontal plane. 5

b) A uniform ladder, of length 70 feet, rests against a vertical wall with which it makes an angle of 45^o, the coefficients of friction between the ladder and the wall and the ground respectively being $\frac{1}{3}$ and $\frac{1}{2}$. If a man, whose weight is one half that of the ladder, ascends the ladder, where will he be when the ladder slips? 5

6- a) A uniform semi-circular wire hangs on a rough peg, the line joining its extremities making an angle of 45^o with the horizontal. If it is just on the point of slipping, find the coefficient of friction between the wire and the peg. 5

b) A uniform rod of weight w is placed with its lower end on a rough horizontal floor and its upper end against an equally rough vertical wall. The rod makes an angle α with the wall and is just prevented from slipping down by a horizontal force P applied at its middle point. Prove that $P = w \tan(\alpha - 2\lambda)$, where λ is the angle of friction and $\lambda < \frac{1}{2}\alpha$. 5

7- a) Find the centriod of the region bounded by the coordinates axes and the circle $x^2 + y^2 = a^2$ which lies in the first quadrant. 5

b) Find the C.M of a hollow right circular cone of semi vertical angle α and height h. 5

8- a) Six equal uniform rods freely jointed at their extremities form a tetrahedron. If this tetrahedron is placed with one face on a smooth horizontal table, prove that the thrust along a horizontal rod is $\frac{w}{2\sqrt{6}}$, where w is the weight of a rod. 5

b) A uniform rod of length 2a rests in equilibrium against a smooth vertical wall and upon a smooth peg at a distance 'b' from the wall. Show that in the position of equilibrium the beam is inclined to the wall at an angle $\text{Sin}^{-1}\left(\frac{b}{a}\right)^{1/3}$ 5