UNIVERSITY OF GUJRAT

A WORLD CLASS UNIVERSITY

(B.A/B.Sc. Part-I) (Old Syllabus & New Syllabus-2016) *Roll No:* ____________

Mathematics B-Course (Paper-I)

Time Allowed : 3 hrs
Max. Marks : 50
Pass Marks : 33%

Attempt **FIVE** Questions in all. Select **TWO** Questions from **Section-A** and **THREE** from **Section-B.**
All Question carry equal marks.

SECTION A

1- a) Let G be any group and let $a \in G$ have order n. Then, for any integer K, a^K if and only if $K = nq$, where q is an integer. *5*

b) If every element of a group G is its own inverse, show that G is abelian. *5*

2- a) Let G be a group and H a subgroup of G. Then the set $aHa^{-1} = \{aha^{-1} : h \in H\}$ is a subgroup of G. *5*

b) Let H and K be two finitie subgroups (of a group G) whose orders are relatively prime, prove that $H \cap K = \{e\}$ *5*

3- a) If $f = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{pmatrix}$, $g = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \end{pmatrix}$ Find $f^2 o g$ *5*

b) Show that an infinite cyclic group has exactly two distinct generators. *5*

SECTION B

4- a) Reduced the matrix $\begin{bmatrix} 6 & 3 & -4 \\ -4 & 1 & -6 \\ 1 & 2 & -5 \end{bmatrix}$ into the echelon form. *5*

b) If A is a square matrix over C, show that $A + (\bar{A})^t, (\bar{A})^t A$ are Hermition matrix. *5*

5- a) Solve the system of equations: *5*

$$2x_1 + x_3 = 1$$
$$2x_1 + 4x_2 - x_3 = -2$$
$$x_1 - 8x_2 - 3x_3 = 2$$

b) Prove that without expansion $\begin{vmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{vmatrix} = 0$ *5*

6- a) Prove that $\begin{vmatrix} 1+a & 1 & 1 & 1 \\ 1 & 1+b & 1 & 1 \\ 1 & 1 & 1+c & 1 \\ 1 & 1 & 1 & 1+d \end{vmatrix} = abcd\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{d} + 1\right)$ *5*

b) Show that $\{1, Sin^2x, Cos2x, Cos^2x\}$ in space of all functions from R to R is linearly independent. *5*

7- a) For an invertible matrix A, $\det A \neq 0$. Prove that $\det A^{-1} = \frac{1}{\det A}$ *5*

b) Show that Transformation $T : R^2 \to R$, $T(x_1, x_2) = x_1 x_2$ is not linear. *5*

8- a) Determine whether the set $\{(1, 1), (3, 1)\}$ is basis for R^2 or not. *5*

b) Find matrix of linear transformation with respect to standard basis

$T : R^3 \to R^3 : T(x_1, x_2, x_3) = (x_2, -x_1, -x_3)$ *5*