

Mathematics General (Paper-II)

Attempt FIVE Questions in all. Select THREE Questions from Section-A and TWO from Section-B.

SECTION-A

1- a) Show that $\lim_{x \rightarrow -\infty} \left(1 + \frac{1}{x}\right)^x = e$ 5

b) Discuss the continuity at $x = 0$ if $f(x) = \begin{cases} \frac{\sin x}{\sin bx} & \text{if } x \neq 0 \\ \frac{b}{a} & \text{if } x = 0 \end{cases}$ 5

2- a) Let $f(x) = |x| + |x - 1|$. Show that the function f is continuous for every value of x but is not differentiable at $x = 0$ and $x = 1$. 5

b) If $f(x) = (\sin x)^{\cos x} + (\cos x)^{\sin x}$. Find $f'(x)$, ($\sin x > 0$, $\cos x > 0$) 5

3- a) If $y = e^{m \arcsin x}$ Show that $(1 - x^2) y^{(n+2)} - (2n + 1) x y^{(n+1)} - (n^2 + m^2) y^{(n)} = 0$ 5

b) Let $f(x, y) = \begin{cases} \frac{x^3 + y^3}{x^2 + y^2} & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{if } (x, y) = (0, 0) \end{cases}$ Examine the continuity at $(0, 0)$.
 Do $f_x(0, 0)$ and $f_y(0, 0)$ exist? 5

4- a) If $x > 0$, prove that $x - \ln(1 + x) > \frac{x^2}{2(1 + x)}$ 5

b) Evaluate $\lim_{x \rightarrow 0} \left(\frac{1}{x \arcsin x} - \frac{1}{x^2} \right)$ 5

5- a) Use mean value theorem to show that $(1 + x)^a > 1 + ax$ where $a > 1$ and $x > 0$. 5

b) Find Maclaurin Series of the function $f(x) = \tan x$ 5

SECTION-B

6- a) Find a reduction formula for $\int x^m (\ln x)^n dx$, $m \neq -1$ and n is an integer greater than 1. 5

Hence evaluate $\int x^3 (\ln x)^2 dx$. 5

b) Evaluate $\int \frac{1}{(e^x - 1)^2} dx$. 5

7- a) Evaluate $\int \frac{\cot x}{1 + \sin x} dx$. 5

b) Evaluate $\lim_{n \rightarrow \infty} \left\{ \left(1 + \frac{1}{n}\right) \left(1 + \frac{2}{n}\right) \dots \left(1 + \frac{n}{n}\right) \right\}^{1/n}$ 5

8- a) Show that $\int_0^{\pi/4} \frac{\sec^2 \theta dx}{\tan x - \tan \theta}$, $\theta > \frac{\pi}{4}$ 5

b) Evaluate $\int_0^{\pi/3} \sin^2 6x \cos^4 3x dx$. 5