UNIVERSITY OF GUJRAT

A WORLD CLASS UNIVERSITY

(B.A/B.Sc. Part-II) (Old & New Syllabus)

Mathematics General (Paper-III)

Roll No: _____

Time Allowed : 3 hrs
Max. Marks : 50
Pass Marks : 33%

Attempt **FIVE** Questions in all. Select **THREE** questions from **Section-A**, and **TWO** from **Section-B**.

SECTION – A

1- a) Find equations of asymptotes for the curve $r(e^{\theta} - 1) = a(e^{\theta} + 1)$. 5

b) Find the relative maxima and minima of y if $r = 1 - \cos\theta$. 5

2- a) In the cycloid $x = a(t + \sin t)$, $y = a(1 - \cos t)$. Prove that $\ell = 4a\cos\left(\frac{t}{2}\right)$ at any point on the curve. 5

b) Find the area of the region bounded by the graph of the parabola $y^2 = 4ax$ and its latus-rectum. 5

3- a) If $U = \arctan\left(\frac{x^3 + y^3}{x - y}\right)$ Show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$. 5

b) If $f(x, y) = 0$ and $\phi(y, z)$, show that $\frac{\partial f}{\partial y}\cdot\frac{\partial \phi}{\partial z}\cdot\frac{\partial z}{\partial x} = \frac{\partial f}{\partial z}\cdot\frac{\partial \phi}{\partial y}$ 5

4-a) Find the volume of the torus generated by revolving the disc $x^2 + y^2 = 4$ about the line $x = 3$ 5

b) The area bounded by the parabola $y^2 = 4ax$ and its latus-rectum is revolved about the tangent at the vertex. Find the area of the surface of the reel thus obtained. 5

5- a) Evaluate $I = \int_D \int \frac{dx\,dy}{x^2 + y^2}$ By changing to polar coordinates where D is the region in the first quadrant between the circles $x^2 + y^2 = a^2$ and $x^2 + y^2 = b^2$, $0 < a < b$. 5

b) Evaluate $I = \int_0^4 \int_0^{4-x} \int_0^{4-x-y} dz\,dy\,dx$ 5

SECTION – B

6-a) The direction cosines ℓ, m, n of two straight lines are given by the equations
$\ell + m + n = 0$ and $2\ell m + 2\ell n - mn = 0$. Find measure of the angle between them. 5

b) Find an equation of the plane through (5, -1, 4) and perpendicular to each of the plane
$x + y - 2z - 3 = 0$ and $2x - 3y + z = 0$. 5

7- a) Show that the straight line $\frac{x}{-1} = \frac{y+1}{2} = \frac{z-2}{-5}$ and the plane $3x + 4y - 2z = 22$ have a unique point of intersection. Find the point of intersection. 5

b) Show that the shortest distance between the straight lines
$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$ is $\frac{1}{\sqrt{6}}$ 5

8- a) Transform the equation $x^2 + y^2 - z^2 = 9$ into spherical coordinate. 5

b) Find an equation of the tangent plane to the sphere $x^2 + y^2 + z^2 - 4x + 2y - 6z = 0$ at the point P(3, 2, 5) 5