UNIVERSITY OF GUJRAT
A WORLD CLASS UNIVERSITY

(B.A / B.Sc. Part-II)(Old & New Syllabus-2016)
Mathematics A-Course (Paper-IV)

Roll No: ________

Time Allowed : 3 hrs
Max. Marks : 50
Pass Marks : 33%

Attempt **FIVE** Questions in all, selecting **THREE** questions from **Section-A**, and **TWO** from **Section-B.**

SECTION-A

1. a) Find equations of asymptotes for the curve $\mathbf{r} = a \sec\theta + b\tan\theta$. *5*

b) Show that the radius of right circular cylinder of greatest curved surface which can be inscribed in a given cone is half that of cone. *5*

2. a) Find the position and nature of multiple points on the curve $x^3 + 2x^2 + 2xy - y^2 + 5x - 2y = 0$. *5*

b) Find the radius of curvature at any point of the curve $\mathbf{r}^n = a^n \sin n\theta$. *5*

3. a) If $u = f(\mathbf{r})$ where $\mathbf{r} = \sqrt{x^2 + y^2}$ prove that $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} = f''(\mathbf{r}) + \dfrac{1}{\mathbf{r}} f'(\mathbf{r})$. *5*

b) Find the % error in the area of ellipse when an error of 1% is made in measuring each of the semi major and semi minor axes of the ellipse. *5*

4. a) Show that the sphere $x^2 + y^2 + z^2 = 18$ and cone $x^2 + z^2 = (y-6)^2$ are tangent along their intersection. *5*

b) Find the extrema of the following $f(x, y) = \dfrac{1}{x} + xy - 8/y$. *5*

5. a) Maximize $z = 500x_1 + 300x_2$ with the conditions *5*

$$x_1 + 2x_2 \le 16$$
$$2x_1 + x_2 \le 11 \qquad x_1, x_2 \ge 0.$$
$$3x_1 + x_2 \le 15$$

b) Minimize $Z = 2x_1 + 3x_2$ *5*

By simplex method
subject to the conditions

$$2x_1 + x_2 \ge 8$$
$$x_1 + x_2 \ge 6$$
$$x_1 + 2x_2 \ge 9$$
$$x_1, \quad x_2 \ge 0.$$

SECTION – B

6. a) Find the area of region bounded by the curve $xy^2 = 4(2 - x)$ and the y-axis. *5*

b) Find the length in one quadrant of the curve $\left(\dfrac{x}{a}\right)^{2/3} + \left(\dfrac{y}{b}\right)^{2/3} = 1$. *5*

7. a) Evaluate $\displaystyle\int_1^2 \int_0^{y^{3/2}} \frac{x}{y^2}\, dx\, dy$. *5*

b) Find the volume of tetrahedron bounded by coordinate planes and the plane $\dfrac{x}{a} + \dfrac{y}{b} + \dfrac{z}{c} = 1$. *5*

8. a) The area bounded by the parabola y2 = 4ax and its latus rectum is revolved about the tangent at the vertex. Find the area of the surface of the reel thus obtained. *5*

b) Find the volume of a right circular cone having base radius **r** and height h. *5*