UNIVERSITY OF GUJRAT

(B.A/B.Sc. Part-I) (Old Syllabus & New Syllabus-2016)

Mathematics A-Course (Paper-II)

Roll No: ____________

Time Allowed : 3 hrs
Max. Marks : 50
Pass Marks : 33%

Attempt **FIVE** Questions in all. Select **TWO** Questions from **Section-A** and **THREE** from **Section-B.**

SECTION-A

1- a) Prove that $\left|\dfrac{az + b}{bz + \bar{a}}\right| = 1$ for $|Z| = 1$ 5

b) $e^{z_1} = e^{z_2}$ if and only if $z_1 - z_2 = 2K\pi i$ where K is any integer. 5

2- a) Separate $(\alpha + i\beta)^{P+iq}$ into real and imaginary parts. 5

b) Prove that $\cos 4\theta = 8\cos^4\theta - \cos^2\theta + 1$ 5

3- a) Prove that $\tan^{-1}\left(\dfrac{x + iy}{x - iy}\right) = \dfrac{\pi}{4} + \dfrac{i}{2}\ln\left(\dfrac{x + y}{x - y}\right)$ $x > y > 0$ 5

b) Evaluate the sum of infinite series $1 + C\cos\theta + \dfrac{C^2}{2!}\cos 2\theta + \dfrac{C^3}{3!}\cos 3\theta + \dots$ 5

SECTION-B

4- a) Find the angle of intersection of the given curve. The parabola $y^2 = 4ax$ and $x^2 = 4ay$ at the point other than (0, 0). 5

b) Identify and graph the given polar equation $r = \dfrac{4}{1 + \cos\theta}$ 5

5- a) Find the Pedal equation of the given curve $r = a + b\cos\theta$ 5

b) Prove that an equation of the normal to the asteroid $x^{2/3} + y^{2/3} = a^{2/3}$ can be written in the form $x\sin t - y\cos t + a\cos 2t = 0$, t being parameter. 5

6- a) A variable in two adjacent positions has direction cosines ℓ, m, n and $\ell+\delta\ell, m + \delta m, n + \delta n$. Show that measure of the small angle $\delta\theta$ between the two positions is given by $(\delta\theta)^2 = (\delta\ell)^2 + (\delta m)^2 + (\delta n)^2$ 5

b) Find an equation of the plane through (5, -1, 4) and perpendicular to each of the planes $x + y - 2z - 3 = 0$ and $2x - 3y + z = 0$ 5

7- a) Find the condition that the straight line $x = mz + a$, $y = nz + b$ may lie in the plane $Ax + by + cz + D = 0$ 5

b) Show that the shortest distance between the straight lines $\dfrac{x - 1}{2} = \dfrac{y - 2}{3} = \dfrac{z - 3}{4}$ and $\dfrac{x - 2}{3} = \dfrac{y - 4}{4} = \dfrac{z - 5}{5}$ is $\dfrac{1}{\sqrt{6}}$ 5

8- a) Express the given equation in cylindrical and spherical coordinates $x^2 - y^2 - z^2 = 1$ 5

b) Show that the two circles $x^2 + y^2 + z^2 = 9$, $x - 2y + 4z - 13 = 0$ and $x^2 + y^2 + z^2 + 6y - 6z + 21 = 0$, $x + y + z + 2 = 0$ lie on the same sphere. 5