UNIVERSITY OF GUJRAT

A WORLD CLASS UNIVERSITY

(B.A/B.Sc. Part-I) (Old Syllabus & New Syllabus-2016) *Roll No:* _____

Mathematics General (Paper-I)

Time Allowed : 3 hrs
Max. Marks : 50
Pass Marks : 33%

Attempt **FIVE** Questions in all. Select **TWO** Questions from **Section-A** and **THREE** from **Section-B.**

Section-A

1- a) Solve the equation $x^{12} - 1 = 0$ and find which of its roots satisfy the equation $x^4 + x^2 + 1 = 0$. 5

b) If $\text{Sin}(\theta + i\varphi) = \cos\alpha + i \text{ Sin}\alpha$ show that $\cos^2\theta = \pm \text{Sin}\alpha$ 5

2- a) Show that $\text{Tan}^{-1} z = \frac{1}{2i} \log\left(\frac{1 + iz}{1 - iz}\right)$. 5

b) If $\log \text{Sin}(x + iy) = u + iv$ show $e^{2y} = \frac{\text{Cos}(x - v)}{\text{Cos}(x + v)}$ 5

3- a) Prove that $\frac{\sin 5\theta}{\sin\theta} = 16 \text{ Cos}^4\theta - 12 \text{ Cos}^2\theta + 1$. 5

b) If $a^{\alpha + i\beta} = (x + iy)^{p + iq}$ $a > 0$ Then show that $\log_a (x^2 + y^2) = \frac{2(\alpha\rho + \beta q)}{\rho^2 + q^2}$ 5

Section-B

4- a) Show every Hermitian matrix can be written as A + iB where A is real and symmetric and B is real and skew symmetric. 5

b) Without expansion show. 5

$$\begin{vmatrix} \frac{1}{2!} & 1 & 0 \\ \frac{1}{3!} & \frac{1}{2!} & 1 \\ \frac{1}{4!} & \frac{1}{3!} & \frac{1}{2!} \end{vmatrix} = 0$$

5- a) Find the rank of matrix 5

$$\begin{bmatrix} 1 & 3 & 1 & -2 & -3 \\ 1 & 4 & 3 & -1 & -4 \\ 2 & 3 & -4 & -7 & -3 \\ 3 & 8 & 1 & -7 & -8 \end{bmatrix}$$

b) Find the matrix of linear transformation from R^3 to R^3 with respect to standard basis of R^3

$T(x_1, x_2, x_3) = (x_1 + x_2, -x_1 - x_2, x_3)$ 5

6- a) Solve by Gauss Jordan method 5

$$3x_1 + 2x_2 + 4x_3 = 7$$
$$2x_1 + x_2 + x_3 = 4$$
$$x_1 + 3x_2 + 5x_3 = 3$$

b) If A and B are 3x3 matrices such that $\det(A^2B^2) = 108$ and $\det(A^3B^2) = 72$, find $\det(2A)$ and $\det(B^{-1})$. 5

7- a) Find equation of tangent and normal to the curve $c^2(x^2 + y^2) = x^2y^2$ at point $\left(\frac{c}{\cos\theta}, \frac{c}{\sin\theta}\right)$ 5

b) Find pedal equation of the curve $r^m = a^m \cos m\theta$. 5

8- a) Find the point at which cardiod $r = 1 + \text{Sin}\theta$ has vertical tangents. 5

b) Sketch the cardiod $r = a(1 - \text{Cos}\theta)$, $a > 0$. 5