UNIVERSITY OF GUJRAT

A WORLD CLASS UNIVERSITY

(B.A/B.Sc. Part-II)(Old & New Syllabus-2016)

Mathematics B-Course (Paper-III)

Roll No: _____

Time Allowed : 3 hrs
Max. Marks : 50
Pass Marks : 33%

Attempt **FIVE** Questions, selecting **TWO** questions form **Section-A,** and **THREE** from **Section-B.**

SECTION – A

1- a) Prove that $9|10^n + 3.4^{n+2} + 5$ for every integer $n \geq 1$. *5*

b) If for a, b, c ∈ z, a|b and a|c, then for any integer x and y, a|bx + cy. *5*

2- a) Show that there are infinitely many primes. *5*

b) If a, b, c are positive integers, then (ac, bc) = c (a, b). *5*

3- a) Find the general solution of the diaphontine equation 2x + y = 1. *5*

b) Find the g.c.d of 1769 and 2378 and express it as a linear combination of these two numbers. *5*

SECTION – B

4- a) Define a topological space and write topologies of 2, 3, 5 and 8 elements on the set x = {a, b, c}. *5*

b) Define closed sets in a topological space and prove the following: *5*

i) Union of finite number of closed sets is a closed set.

ii) Intersection of any number of closed sets is a closed set.

5- a) If X is a topological space and A ⊆ X, define closure of A and prove that A is closed if and only if A = cℓ(A). *5*

b) A subset A of a topological space is closed if and only if it contains all of its limit points. *5*

6- a) If (X, d) is a metric space, then show that $d_1(x_1y) = \frac{2\,d(x,y)}{1 + 2\,d(x,y)}$ is also a metric on X. *5*

b) Every non-empty subset of a discrete metric space is an open set. *5*

7- a) Open sphere in a metric space is an open set. *5*

b) If A and B are two subsets of a metric space x, then Ext (A ∪ B) = Ext A ∩ Ext B. *5*

8- a) Show that (1, -1, 0), (2, -1, -2), (1, -1, -2) is a basis of R^3. Find an orthonormal basis of R^3 using Gram-Schmidt orthonormal process. *5*

b) A square matrix A is orthogonal if and only if the columns of A form an orthonormal set. *5*