

Mathematics General (Paper-IV)

Time Allowed : 3 hrs
 Max. Marks : 50
 Pass Marks : 33%

Attempt FIVE Questions in all, Select TWO questions from Section-A, and THREE from Section-B.

SECTION-A

1. a) Test the series for convergence or divergence $\sum_{n=1}^{\infty} \frac{2^n}{3^n + 1}$ 5

b) Use the appropriate test to determine the convergence or divergence of the series. $\sum_{n=1}^{\infty} \frac{(n!)^2 2^n}{(2n+2)!}$ 5

2. a) Test the series for i) Absolute Convergence ii) Conditional Convergence iii) Divergence

$\sum_{n=1}^{\infty} (-1)^{n-1} \left(\frac{n+2}{3n-1} \right)^n$ 5

b) Find the radius of convergence $\sum_{n=1}^{\infty} \frac{1+(-1)^n}{n} x^n$ 5

3. a) Use Newton's Raphson Method to find a positive real root of the equation up to four places of decimal

$e^x = 2x + 21$, $x_0 = 3$ 5

b) Use Bisection Method to find a real root of $f(x) = e^x - 3x = 0$ up to five iteration's 5

SECTION-B

4. a) Solve $e^x \left(1 + \frac{dy}{dx} \right) = x e^{-y}$ 5

b) Solve $(1 + \ln xy) dx + \left(1 + \frac{x}{y} \right) dy = 0$ 5

5. a) Find an equation of orthogonal trajectories of the curve $y^2 = x^2 + cx$ 5

b) Find the general solution $(D^2 - 2D - 3)y = 2e^x - 10\sin x$ 5

6. a) Solve by the method of undetermined coefficient $2y'' + 3y' + y = x^2 + 3\sin x$ 5

b) Solve $x^2 \frac{d^2y}{dx^2} - 3x \frac{dy}{dx} + 5y = x^2 \sin(\ln x)$ 5

7. a) Solve $\frac{d^2y}{dx^2} + y = \sec^3 x$ 5

b) Solve $(x^2 + 1) \frac{dy}{dx} + 2xy = 4x^2$ 5

8. a) Maximize 5

$z = 7x_1 + 5x_2$ such that

$x_1 + 2x_2 \leq 6$

$4x_1 + 3x_2 \leq 12$

$x_1 \geq 0, x_2 \geq 0$

using the Graphical Method

b) Maximize $z = 3x_1 + 2x_2$ with 5

$x_1 + 2x_2 \leq 6$

$2x_1 + x_2 \leq 8$

$-x_1 + x_2 \leq 1$

$x_2 \leq 2$

$x_1 \geq 0, x_2 \geq 0$ using the Simplex Method