Measurement of Educational Disparities in Punjab (Pakistan): An Analysis from MICS's Micro-data

Tanveer Ahmed Naveed*,David Gordon**, & Sami Ullah***

Abstract

The primary objective of this research study is to calculate education index at household level and to estimate the education disparities for all districts of Punjab by using MICS microdata and to devise empirical analysis for investigating the relationship between educational attainment, income and educational disparities in Punjab. We have used educational attainment, reading-ability and writing-ability as indicators to estimate education index at household level. While Gini-coefficient, Atkinson index and Generalised entropy are used to estimate the educational disparities. The districts, Rawalpindi and Chakwal, score highest education index value 0.78 and 0.74 respectively. In Punjab, most unequal education distribution is observed in district Rajanpur while least unequal distribution in Chakwal, with Gini-coefficient 0.51 and 0.21 respectively. The districts with lower education (like Rajanpur, DG Khan, RY Khan) are facing higher rural-urban education gaps. The estimates show the strong correlation between income and education indices. Income (permanent) affect positively and education disparities affect inversely to the education attainment. Further, education disparity (education Gini) decreases as average education attainment (education index) rises. It has also been observed that the education disparities in low income districts is most likely to be worse than that of the high income districts of Punjab. Furthermore, a positive association between income inequalities and education disparities exists. Moreover, the districts with higher gender disparities are facing higher education inequalities with in society, while districts with less gender related education inequalities are enjoying more equal distribution of education. Likewise, at higher rural-urban differences educational Gini would also be higher, as rural-urban gap reduces and move towards equality at start education disparities reduces at slower rate but after a point it decline sharply. This study also concludes that education disparities are higher among Saraiki speaking districts (Southern Punjab).

Key words: Education index, Education disparities, Gini-coefficient, Atkinson's index
JEL Classifications: D63, I24, I21, I25

Introduction

Traditionally income variables are been considered the central indicators to gauge the level of well-being or prosperity of any society. While many scholars have put emphasis on insertion of the non-economic dimensions as well for the measurement of wellbeing; like health, education. Particularly, the education plays a persistent role in well-being measurement, human capital formation and assessment of quality of human life (Sen, 1988; Acemoglu and Angrist, 2001; Stiglitz et al., 2010; Oreopoulos and Salvanes, 2011). The education is imperative mean to strengthen the human capital, which is considered leading element for economic growth and development (Locus, 1988; Becker et al., 1990; Rebelo, 1991; Lin, 2015). Moreover, education is also necessary for poverty reduction, conflict resolution and gender equality (Antoninis, Delprato, Aaron Benavot, 2016). Education equality would be one of the most important factors of economic harmony, and faster growth. That's a reason, why one of the new Sustainable Development Goals (SDGs), SDG-4 and target 4.5 exclusively emphasises on the inevitability of ensuring equal access to education at all levels.

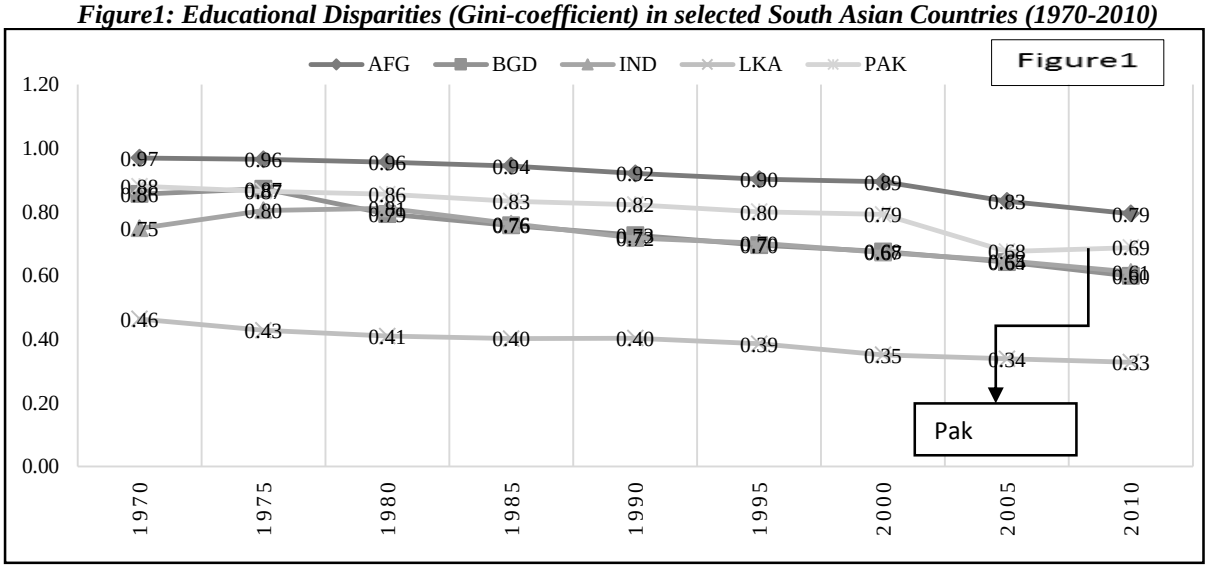
* PhD Scholar/Assistant Professor, University of Gujrat, Gujrat
**Professor/Director, Bristol Poverty Institute, University of Bristol, UK
***Assistant Professor, University of Gujrat, Gujrat

The SDG-4; "*Ensure inclusive and equitable quality education and promote lifelong learning opportunities for all*". This goal represents the faith that inclusive quality education is one of the most dominant and powerful vehicles for sustainable development. This goal certifies the complete free primary and secondary schooling, equal access to affordable vocational training, and higher education, regardless of gender and income groups, by 2030. Educational inequality reduction would enlarge the progressive impact on attainment of other SDGs and helpful in reducing probability of conflicts (UNESCO, 2014). By considering education vital for economic progress and social achievements, the development economists have emphasis on measurement of reliable estimate of educational attainment that could be acceptable across the country and over the time (for example Barro and Lee, 1993; Cohen and Soto, 2007; Morrisson and Murtin, 2009; Barro and Lee, 2013; Jorda and Alonso, 2017).

Since 1990, UNDP has been using gross school enrolment ratio of children and adult literacy rate jointly as a proxy measure for access to knowledge. Which latter on replaced, since 2010, with the indicators mean years of schooling (MYS) for adults with twenty five years and above age, and expected years of schooling (EYS) for school entering age children. Over the time, the indicator years of schooling also used to analyse educational inequalities in different countries (Ram, 1990; World Bank, 2005), to estimate the schooling distribution at national level (Thomas, Wang, & Fan, 2001; Castello-Climent, 2008; Meschi & Scervini, 2014). For estimation of disparities, more than a hundred approaches are suggested by literature on measurement of disparity postulates (Maio, 2007). Various studies have used Gini-coefficient for estimating disparities in education (e.g. Ter Weele, 1975; Maas and Criel, 1982; Rosthal,1978; Sheret, 1988; Lopes et al., 1998, 1999; Thomas et al., 2001).

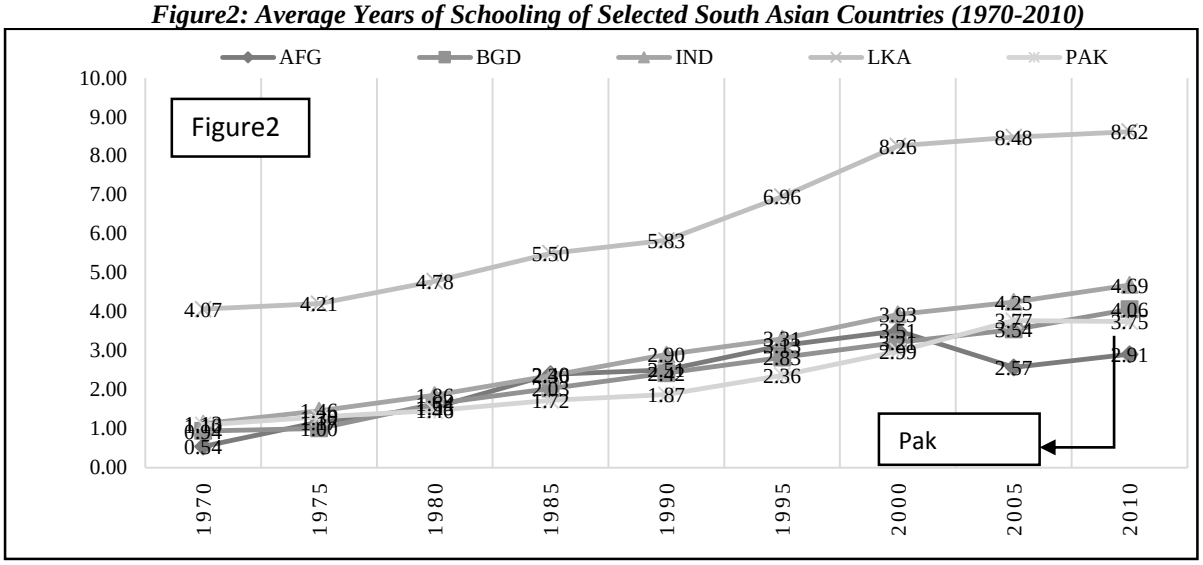
While talking about Pakistan, it is ranked at 147 out of 188 countries for human development index (HDR, 2016). Moreover, Pakistan in the map of world is facing severe social, regional and income inequalities. Pakistan is ranked at 143 position out of 144 countries on the pedestal of Global Gender Gap index by having value of 0.546 score, the worst position in South Asian countries (World Economic Forum, 2017). As for as, Gender Inequality Index (GII) is concerned, Pakistan is ranked at 133 out of 151 countries with the value of index 0.541 (HDR, 2017). According to Economic Survey of Pakistan (2011-2012) only 21.5 percent females participate in labour force.

Figure1: Educational Disparities (Gini-coefficient) in selected South Asian Countries (1970-2010)



Source: Barro, R. and Lee, J.W., August 2014

Figure2: Average Years of Schooling of Selected South Asian Countries (1970-2010)



Source: Barro, R. and Lee, J.W., August 2014

Among the South Asian counties, Pakistan's performance in education is not reasonably satisfactory (see Figure1), moreover, education disparities are higher and significant efforts towards education disparities reduction are not witnessed (see Figure 2). Pakistan is consisted upon five provinces. Where contribution of Punjab in total population (of Pakistan) is more than 56 percent and 52 percent share in GDP of Pakistan. Punjab is consisted upon nine divisions, and 36 districts. In Pakistan, regional disparities exist among the provinces, with in provinces and as well as among the districts (HDR, 2003). Human Development Index (HDI) for district Jhelum is 0.703 comparing to district Dera Bhugti with HDI value 0.285 which is lowest in Pakistan. In Punjab, Jhelum stands at first and Muzaffargarh at last on the pedestal of HDI ranking. Talking about, in particular, education disparities; on the basis of average years of schooling district Rajanpur is more deprived by having 4.41 for male and 1.95 for female contrary to outperformer Rawalpindi having 8.5 for male and Lahore 7.34 for female. The district Rajanpur featured by highest education disparities with a value of Gini-coefficient 0.824 for female and 0.61 for male. Moreover, district Chakwal is ranked at lowest for education disparities with 0.26 Gini-coefficient, while district Lahore for female inequalities with Gini value 0.42 (Saeed and Yibing, 2016).

Taking 2030 agenda at top note, estimating reliable education measures, calculating education disparities and investigating factors of education disparities at household level is imperative for suggesting effective policy options, and finding the targets of SDGs in true spirit. The main objectives of this research study are (a) to calculate education index at household level by using MICS microdata for all districts of Punjab, (b) to estimate the education disparities for all districts of Punjab, and (c) to devise empirical analysis for investigating the relationship between educational attainment, income and educational disparities in Punjab. The next section, data and methodology, is divided into two parts. First part explains the methodological description of measurement of educational index at household level. Second section describes the methodological debate on measurement of education disparities e.g. Lorenz curves, Gini-coefficient, Theil index and generalized entropy. The last section explains the results of the study and suggests possible policy options for education disparities reduction.

Data and Sampling

Multiple Cluster Indicator Survey (MICS) is international household survey initiative organized by UNICEF to fill the data gaps and to highlight the children and gender related issues. UNICEF is providing strategic, technical and methodological research assistance from last two decades to the different countries of the world. The data of MICS surveys is considered most

reliable on the household characteristics, gender, health, education, child care and other socio-economic indicators. Pakistan Bureau of Statistics has designed the sample for MICS survey for at provincial level, for Punjab, that includes 9 divisions and 36 districts. The sample size of MICS 2014 is around 38,405 households. Within each district, population was divided into rural and urban strata, and from each strata sample has been selected.

Table 1: MICS Sample by Locality and Gender of Head of Household[1]

Area	Male	Female	Total
Rural	22,422	1,819	**24,241**
Urban	12,919	1,245	**14,164**
Total	**35,341**	**3,064**	**38,405**

Measuring Education Index

By considering education vital for economic progress and social achievements, the development economists have emphasis on measurement of reliable estimate for educational attainment that could be acceptable across the country and over the time (for example Barro and Lee, 1993; Cohen and Soto, 2007; Morrisson and Murtin, 2009; Barro and Lee, 2013; Jorda and Alonso, 2017).

Since 1990, UNDP have been publishing Human Development Index annually, that gauges the average success of a nation in achieving long and health life, access to knowledge and decent living standard for it citizens. Where access to knowledge have been measured by mean years of schooling (MYS) for adults with twenty five years and above age, and expected years of schooling (EYS) for school entering age children. The mean years of schooling is "*the average number of years of education received by people ages 25 and older in their lifetime based on education attainment levels of the population converted into years of schooling based on theoretical durations of each level of education attended*" (UNDP, 2010. p.224) While expected years of schooling is *"the number of years of schooling that a child of school entrance age can expect to receive if prevailing patterns of age specific enrolment rates were to stay the same throughout the child's life"* (UNDP, 2010. p.223).

The education index, estimated by jointly mean years of schooling and expected years of schooling, is considered good proxy variable for measurement of educational attainment and gaining knowledge. Lack of micro level data on exact number of completed years of schooling is one of the major problems in measurement of MYS. According to level of educational attainment, mostly, individuals are grouped into four educational categories like; no schooling or pre-primary, primary level of education, secondary, and tertiary level. Further, information about completion of educational cycle is essential, that's is most of time not available. So the variable educational attainment is a categorical whose mean years of schooling is calculated by taking weighted average of the country specific official duration of each educational cycle.

In this paper we have calculated aggregated education index comprising upon four indicators; first, mean years of schooling 25 and above age (an household level continuous variable), second the expected years of schooling below 25 years age, and to incorporate the quality of education into the aggregated index we have used additional two indicators (third and fourth) the reading-ability and writing-ability. As this study has used the MICS micro data for Punjab, so measurement of education index is limited to available information in the survey. For mean years of schooling and expected years of schooling at household level this study has used following three questions from MICS Punjab 2014 survey data, the first question (ED3); *has the member of family attended school or pre-school?* Second question (ED4A); *what highest level of school has attended?* Third question (ED4B); *what is the highest grade completed this level?*

[1] As per MICS 2014, total 41,413 household heads were selected for household questionnaires, out of that 38405 were interviewed rest either not available or not cooperated. Moreover, for women related issues 53,668 women with age 15-49 and for children (under five year's age) related issues 27,495 mothers/caretakers were interviewed respectively.

Second question explains the individual belong to which educational attainment group e.g. pre-school, primary, middle, matric, or tertiary level. Third question refers to highest completed grade e.g. according to education system in Pakistan primary education grade is between 1 to 5 years of schooling, middle 1 to 3 years after primary, matric 1 to 2 years after middle, tertiary or higher level 1 to 6 years after matric for complete 16 years of education (a university degree) but this survey do not provide information about post-graduate level education like; MPhil 1 to 2 years after BS or MSC, and PhD 1 to 3 years after MPhil.

The mean years of schooling or MYS designates the average number of finished or completed years of education of a particular country's population aged 25 years and older. This methodology has been adopted from Baroo and Lee (1993, 2010, 2013) and UNESCO-UIS (2013, 2014). To estimate MYS at household level for Punjab (Pakistan) following information is required;

- Distribution of population by age group and highest level of education attained in given year; and
- Official duration of each level of education in Punjab, Pakistan.

Formal method for MYS calculation is as follow;

$$MYS_h = \sum_a \sum_I HS_{al} \times YS_{al} \dots\dots\dots eq(1)$$

Where

MYS_h: is mean years of schooling of each household

HS_{al} Proportion of population in age group a for which level of education I is the highest level attained

YS_{al} Official duration of the level of education I for age group a at the time when this age group was in school

Whereas, expected years of schooling is calculated by adding up specific rates of enrolment by age weighted by the respective amplitude of the age group, formal method is as follow;

$$EYS_a = \sum_a^w n \times {}_nm_x \dots\dots\dots eq(2)$$

Where

$${}_nm_x = \frac{{}_nf_x}{{}_np_x}$$

Where, a is the schooling starting age, w is the upper age limit, and n is age interval. Moreover, ${}_nf_x$ is the number of children between ages x and x+n enrolled in school in particular household, ${}_np_x$ is age specific population between the ages x and x+n in particular household, and ${}_nm_x$ is enrolment rate of children between ages x and x+n in a particular household.

At first step, mean and expected years of schooling at household level is calculated. At second step, both indicators are normalized[2] by using minimum value of 0 and maximum value 16 for both MYS and EYS. So the results of MYS and EYS may be underrated in this study and would not be comparable at district level results of Pakistan HDI Report 2017.

[2] Dimension index = $Actual\ value - minimum\ value / maximum\ value - minimum\ value$

Indicator	Maximum[3] Value	Minimum value
Mean Years of Schooling	16	0
Expected Years of Schooling	16	0
Readability	100	0
Write-ability	100	0

Having years of education (attainment) is meaningless if people are unable to read or write[4], so quality of education is imperative for measurement of education index. OECD[5] is committed to incorporate the quality in measuring education by initiating PISA[6]. Mostly, data on quality indicators for education is not available for developing countries, but according to MICS survey data for Punjab, question WB7, survey enumerators presented a card to the interviewee and asked to read the sentence(s), and recorded the information accordingly. Moreover, the question HL10A also recorded the information about reading-ability and the question HL10C record the information about writing-ability in the MICS survey. The indicators reading-ability and writing-ability are normalized[7] by using minimum value 0 and maximum value 100. Where 0 means no one in home is able to read or write, and 100 means hundred percent household members has reading and writing ability.

Once we have calculated education indicators; MYS, EYS, reading ability and writing ability for each household by using MICS micro data for Punjab, household based education index has been calculated by using Equation 3, where $E^h$is household specific education index, $MYSi^h$is calculated household specific adult mean years of schooling index, $EYSi^h$is expected years of schooling index, $Ra^h$is reading ability household index, and Wa^hhousehold's writing ability index. Moreover $a^h$is the value of indicator of a specific household under discussion.

$$E^h = \left(\frac{1}{4}\right) \times MYS^h + \left(\frac{1}{4}\right) \times EYS^h + \left(\frac{1}{4}\right) \times Ra^h + \left(\frac{1}{4}\right) \times Wa^h \quad h = 1,2,..,K. \qquad Eq(3)$$

Where

$$MYSi^h = \frac{a^h - 0}{16 - 0} \quad h = 1,2,..,K.$$

and

$$EYSi^h = \frac{a^h - 0}{16 - 0} \quad h = 1,2,..,K.$$

and

$$Ra^h = \frac{a^h - 0}{1 - 0} \quad h = 1,2,..,K.$$

and

$$Wa^h = \frac{a^h - 0}{1 - 0} \quad h = 1,2,..,K.$$

[3] Setting the maximum goal posts for education is based upon estimated maximum values of MYS and EYS at household from the MICS survey. There are some households who own maximum 16 MYS and EYS, therefore maximum goal post for both indicators is set at 16 and minimum at 0. Pakistan Human Development Index Report 2017 has measured MYS and EYS at district level and set maximum goal posts for MYS at 10 and EYS at 15, based upon maximum estimated values of Islamabad. It is not possible for a district in Pakistan to score value 15 MYS and 18 EYS, but at household level it is possible for any household that all adults and children would score maximum years of schooling.

[4] By David Gordon, Director, Townsend Centre for International Poverty

[5] OECD (2016), "Executive summary", in PISA 2015 Results (Volume I): Excellence and Equity in Education, OECD Publishing, Paris. DOI: https://doi.org/10.1787/9789264266490-2-en

[6] The Programme for International Student Assessment (PISA), OECD initiative, is an international survey with an aims to assess the global education systems by analysing the skills and knowledge of 15-year-old students. In this survey over a half million 15 years above people agreed to take two hours test. The test was comprised on the subjects; mathematics, collaborative problem solving, reading and financial literacy. The survey has been representing more than 28 million people 15-years old, from 72 countries OECD

[7] Dimension index = $Actual\ value - minimum\ value / maximum\ value - minimum\ value$

Measuring Education Disparities

A wide range of approaches exist for measurement and analysis of education disparities. Literature on disparity measurement explains that there are more than 115 statistical or econometric tests on measurement of disparity postulates (Maio, 2007). The simplest way is known as Lorenz curve. The Lorenz curve displays the cumulative percentage of education index on vertical axis, and cumulative percentage of population on horizontal axis. Degree of inequality is presented by Lorenz curve away from the 45 degree line (also known as education egalitarian line or perfect equality line). Although, no consensus has been emerged yet but Gini-coefficient is considered most popular and extensively cited measure of education disparity. It is based on Lorenz curve framework, and ranged between 0 and 1. Zero mean perfect equality, and one mean perfect inequality.

Various studies have used Gini-coefficient in estimating disparities in education; for example Ter Weele (1975) calculated education Gini-coefficient for African countries by using education finance data, Maas and Criel (1982) estimated disparities by using enrolment, education attainment and education expenditures data for 16 East African economies, Rosthal (1978) investigated education inequalities for USA, Sheret (1988) estimated education Gini by using enrolment data for Papua New Guinea, Lopes et al., (1998, 1999) estimated gini-coefficient for 12 economies by using education attainment data, Thomas et al (2001) calculated education Gini index by using education attainment (years of education) data for population aged 15 and above for 85 economies.

Gini-Coefficient

The most appealing property of the Lorenz curve framework is that it could generate a single summary statistic of education distribution, known as Gini-coefficient. The value of Gini-coefficient lies between 0 and 1. Where 0 mean perfect equality that reflects education is equally shared by all people of society. The deviation of Lorenz curve from the 45 degree line mean higher the value of Gini-coefficient that would reflect higher inequality.

By adopting a formal statistical method, lets by taking x_i on the X-axis, while y_i on Y-axis. Then

$$Gini = 1 - \sum_{i=1}^{N} (x_i - x_{i-1})(y_i + y_{i-1}) \dots\dots\ eq(4)$$

By having N equal intervals on the X-axis, the above equation would be simplified as

$$Gini = 1 - \frac{1}{N} \sum_{i=1}^{N} (y_i + y_{i-1}) \dots\dots\ eq(5)$$

These equations are presented without weights that may be incorporated by using statistical software of choice.

Atkinson Index

One of the most popular welfare based method for inequality measurement is known as Atkinson's Index or Atkinson's inequality measure. This index is determined by the degree of society aversion (this theoretical parameter is represented by epsilon and researcher can decide any positive value for it) to inequality. Higher the value of parameter (epsilon) requires the higher readiness by residents of society to receive reduced share of education in order to have more equal distribution. Atkinson's index has distinctive feature that it can be decomposed into with and between the group disparities (Bellu, 2006).

The Atkinson Index[8] of inequality would be defined as follow;

[8] Atkinson index of inequality played a prominent role in welfare based measures of inequality. The Atkinson Index in a straight line connected to the class of additive Social Welfare Function (SWF).

$$A(\varepsilon) = 1 - \frac{OC}{OB} = 1 - \frac{y_{EDE} * \sqrt{2}}{\bar{y} * \sqrt{2}} = 1 - \frac{y_{EDE}}{\bar{y}} \dots\dots\dots\dots eq\ 6$$

Generalized Entropy

The entropy is measure of disorder, here it means measuring deviation from perfect education equality. The formal presentation of generalised disparity index is as follow;

$$E_{(\alpha)} = \frac{1}{n(\alpha^2 - \alpha)} \sum_i \left[\left\{\frac{y_i}{\bar{y}}\right\}^{\alpha} - 1\right] \dots\dots\dots\dots eq\ 7$$

The equation above defines the class of generalised entropy, depending upon the value allotted to alpha, α , the index above, $E_{(\alpha)}$ may assume different methods. If positive higher value is assigned to alpha, α, the index $E_{(\alpha)}$ would be more sensitive to the upper tail of the education distribution and vice-versa. In principle, the parameter alpha, α, may range from -∞ to +∞. But, in general, alpha is preferred to be positive (non-negative) because if alpha is negative the index would be undefined for zero level of education. The above [9]equation modifies as follow, if: (i) α=0; (ii) α=1.

$$W = \frac{1}{N}\sum_{i=1}^{n} U(y_i) \dots\dots\dots\dots (eq\ 1)$$

Equation 1 represents that social welfare (W) is function of average utility. Conferring to Atkinson utility function (U) can be as follow;

$$U(y_i) = \frac{1}{1-\varepsilon} y_i^{1-\varepsilon} \dots\dots\dots\dots eq\ 2a \qquad \text{Where } \varepsilon \neq 1$$

$$U(y_i) = log y_i \dots\dots\dots\dots eq\ 2b \qquad \text{Where } \varepsilon = 1$$

$$U(y_i) = y_i \dots\dots\dots\dots eq\ 2c \qquad \text{Where } \varepsilon = 0$$

In above equations, epsilon (ε) is the parameter of inequality aversion. If ε *is eqaul* to zero than utility function will be equal to mean education. That mean higher the mean education will lead to higher social welfare. In this case, social welfare function (SWF) has characteristic of having W greater than 0. The demonstration of this condition is as follow;

$$\frac{\partial W}{\partial y_i} = \frac{1}{n}\frac{1-\varepsilon}{1-\varepsilon} y_i^{1-\varepsilon-1} = \frac{y^{-\varepsilon}}{n} > 0 \dots\dots\dots\dots eq\ 3$$

But as ε rises, rise in lower education would be given relatively higher weight in producing SWF. That implies social welfare function must be having second order derivative negative. Atkinson establish this condition as follow;

$$\frac{\partial^2 W}{\partial y_i^2} = -\varepsilon \frac{y^{-\varepsilon-1}}{n} < 0 \dots\dots\dots\dots eq4$$

Looking above discussion, ε reveals the researcher's value judgement, so the equation 1 and 2's exact specification depends upon the value of epsilon (ε).

The Atkinson's inequality measurement method is based on the concept of Equally Distributed equivalent (EDE). More properly, the Equally Distributed Equivalent is that level of education or share of education that, if acquired from the every single person in the education distribution, would qualify the society to stretch the equal level of welfare as actual education level.

The Atkinson Index of inequality would be defined as follow;

$$A(\varepsilon) = 1 - \frac{OC}{OB} = 1 - \frac{y_{EDE} * \sqrt{2}}{\bar{y} * \sqrt{2}} = 1 - \frac{y_{EDE}}{\bar{y}} \dots\dots\dots\dots eq\ 5$$

The equation above, Atkinson Index, illuminates that how much share of education we would be willing to sacrifice in order to attain equal level of education. Here, we have used an expression of EDE in order to adopt operational method for inequality index of Atkinson i.e.

$$U(y_{EDE}) = \frac{1}{1-\varepsilon}(y_{EDE})^{1-\epsilon} \dots\dots\dots\dots eq\ 6$$

Refer to figure above, social welfare in equation 1 must be similar with equation 2 and 6; as follow

$$W = \frac{1}{n}\sum_i \frac{y_i^{1-\varepsilon}}{1-\varepsilon} = \frac{1}{n} n \frac{(y_{EDE})^{1-\epsilon}}{1-\varepsilon} \dots\dots\dots\dots eq\ 7$$

The expression of EDE can be derived as;

$$y_{EDE} = \left(\frac{1}{n}\sum_i y_i^{1-\epsilon}\right)^{\frac{1}{1-\varepsilon}} \dots\dots\dots\dots eq\ 8$$

Supposing, when $\varepsilon = 1$, equation will be as follow;

$$y_{EDE} = \prod_i (y_i)^{\frac{1}{n}} \dots\dots\dots\dots eq\ 9$$

In short, by given any level of education distribution, for different levels of education inequality aversion, the Equalised Distribution Equivalent (EDE) can easily be calculated. The different values of epsilon (ε) will give different y_{EDE}, for example, when $\varepsilon = 0$ than $y_{EDE}$is equal to the average level of education; when $\varepsilon > 0$ than y_{EDE} decreases and A(ε) will be rising. More precisely, suppose with $\varepsilon = 2$, the Atkinson index value is A(2)=0.42 that mean society is prepared to give up 42 percent of the education to achieve equal level of educational distribution or society need to sacrifice 42 percent of education for equality.

[9] For detailed mathematical derivation of class indices reader may see: EASYPol Module 051: Describing income inequality: Theil and Entropy class indexes.

$$E_{(0)} = -\frac{1}{n}\sum_{i} \ln\left\{\frac{y_i}{\bar{y}}\right\} \ldots\ldots\ldots\ldots eq\ 8$$

$$E_{(1)} = \frac{1}{n}\sum_{i} \left\{\frac{y_i}{\bar{y}}\right\} ln\left\{\frac{y_i}{\bar{y}}\right\} \ldots\ldots\ldots\ldots eq\ 9$$

When alpha is equal to zero ($E_{(0)}$), the index is known as mean logarithmic deviation. And when alpha is equal to one (when $E_{(0)}$), the index is known as Theil Index (proposed by Theil in 1967). However, this index requires to replace zero level of education or income to replace with very small arbitrarily. The Generalized Entropy (GE) is one of the widely used and accepted method because it satisfy most of the criteria of good measure of disparities. The value of this measure lies between zero and infinity or one (if normalized). Where zero means perfect equality of education distribution. Mostly, in common practice, 0, 1, and 2 are the values of alpha used. The index is called '***Theil's L'*** or mean 'log deviation', when alpha is assumed equal to zero. The index is called 'Theil's T' index, when alpha value is equal to one. The measure is called coefficient of variation, when alpha value is equal to two.

To sum up, there are a lot of measures of inequality, so one need to have good reasons or at least pragmatic reasons to decide which ones to use. The Gini-coefficient is the most widely used measure of inequality so it is a good idea to use this measure. The Atkinson Index is used by UNDP to calculate their Inequality Adjusted HDI - this provides a good reason to use the Atkinson Index. UNDP had put an effort to use Gini-coefficient for measurement of inequality in human development index distribution (Hicks, 1998) but Atkinson index was considered better measure for three reasons, firstly, its subgroup consistency, secondly, its sensitivity for education inequality towards lower tails of distribution and thirdly, its computational simplicity and mathematical sophistication.

Results and Discussion

The primary objectives of this research study includes (a) to calculate education index at household level by using MICS microdata for all districts of Punjab, (b) to estimate the education disparities for all districts of Punjab, and (c) to devise empirical analysis for investigating the relationship between educational attainment, income and educational disparities in Punjab. This section, results and discussion, would be comprised by three parts, first part explains the descriptive analysis of the education index, second part represents education disparities across the districts in Punjab, and third part explains relationship between educational attainment, income and educational disparities in Punjab.

Educational attainment

The computation of education index at household level is comprised by four indicators; mean years of schooling, expected years of schooling, reading-ability and writing-ability indices at household level. The figure 2 below portrays, overall, that districts in southern Punjab (like; Rajanpur, DG Khan, RY Khan and Muzaffargarh) are low performer in education and stayed at bottom, while districts situated in central Punjab and in north (like Rawalpindi, Chakwal, Lahore, Jhelum and Gujrat) are relative good in performer in education.

Figure 2: District wise education index of Punjab

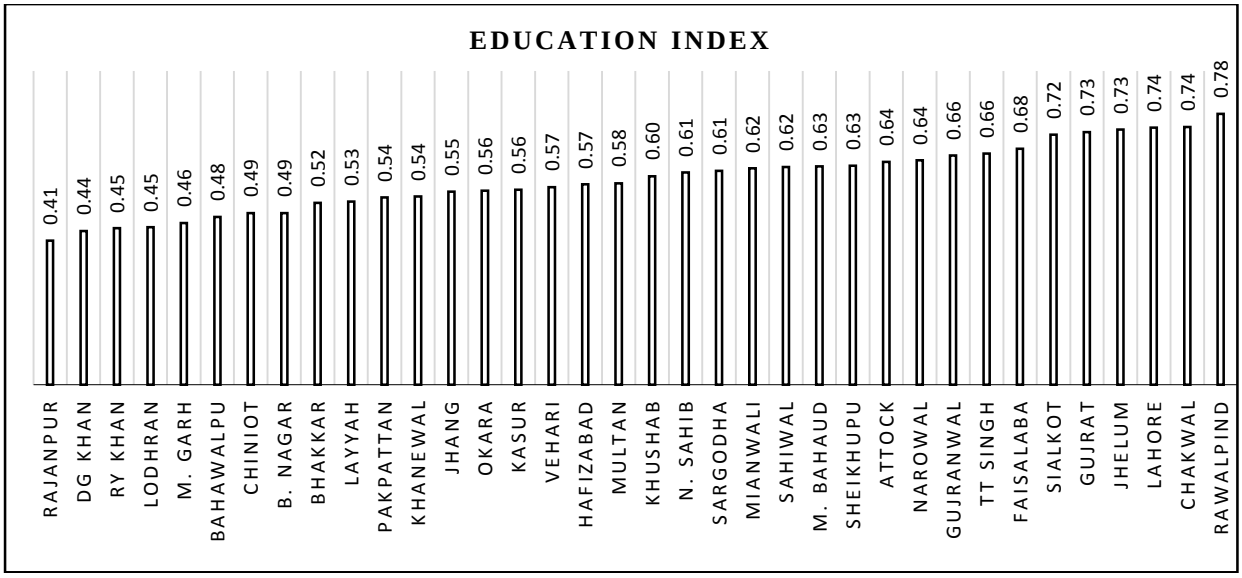
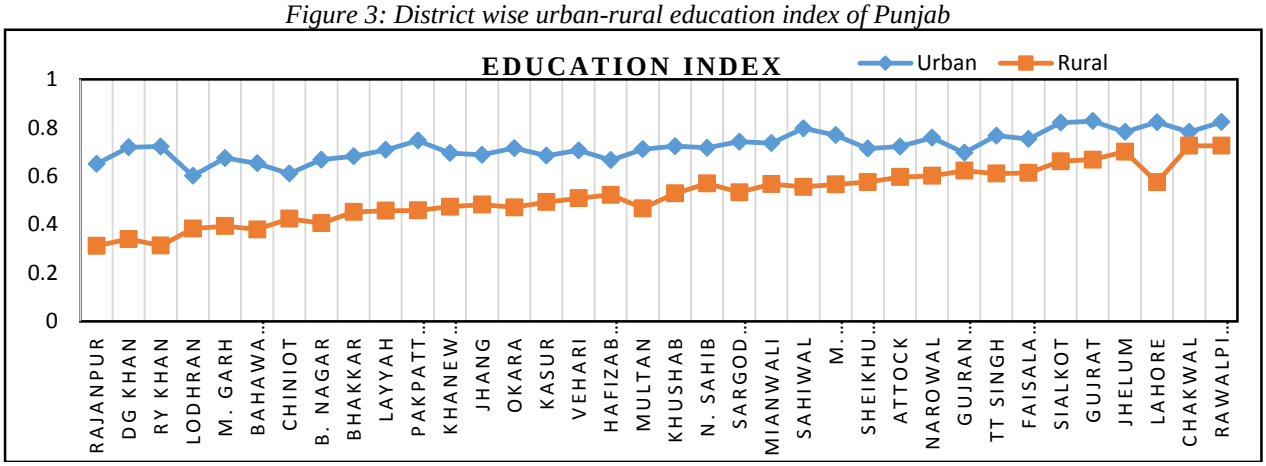


Figure 3: District wise urban-rural education index of Punjab



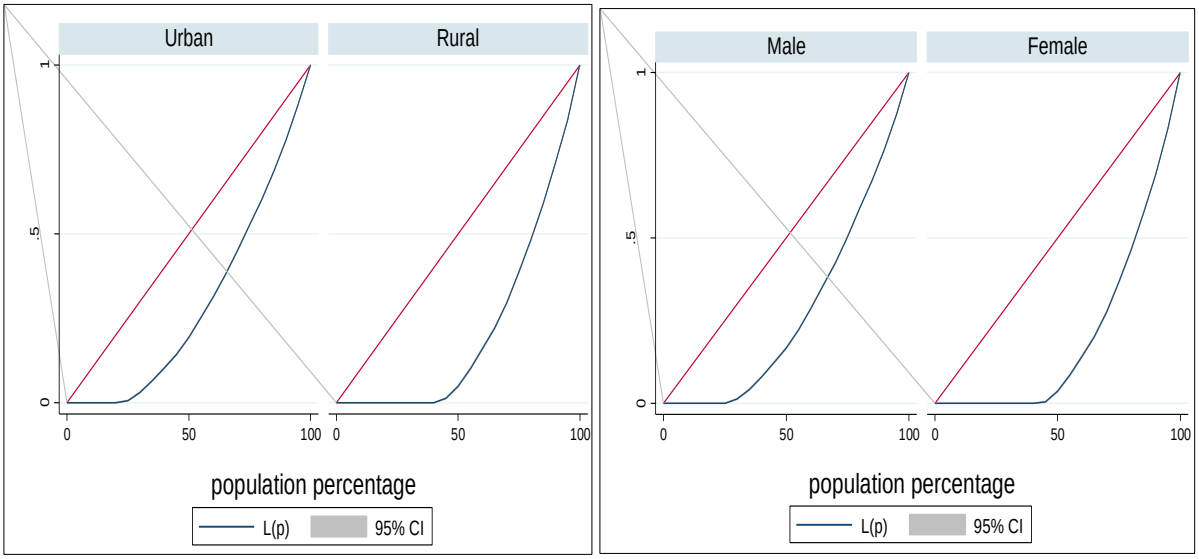
The figure 3 illustrates the district wise rural-urban education attainment gap. Urban population has higher access to education facilities comparing to its rural counterpart. Education for all, regardless to gender or locality, is the SDG goal; many districts are much away from the target of providing equal education opportunities, particularly, it would be considered government system failure if compulsory education is not attained in coming years. Only fewer districts are performing better in squeezing the gap between rural-urban educational attainments. It explains that rural-urban education attainment gap reduces with the higher education index; like Chakwal and Jhelum. In general, the districts with lower education (like Rajanpur, DG Khan, RY Khan) are facing higher rural-urban education gap. Particularly, district Chakwal, Jhelum and Gujranwala are with lowest rural-urban education differences.

Education disparities in Punjab

Education Lorenz Curves of districts of Punjab

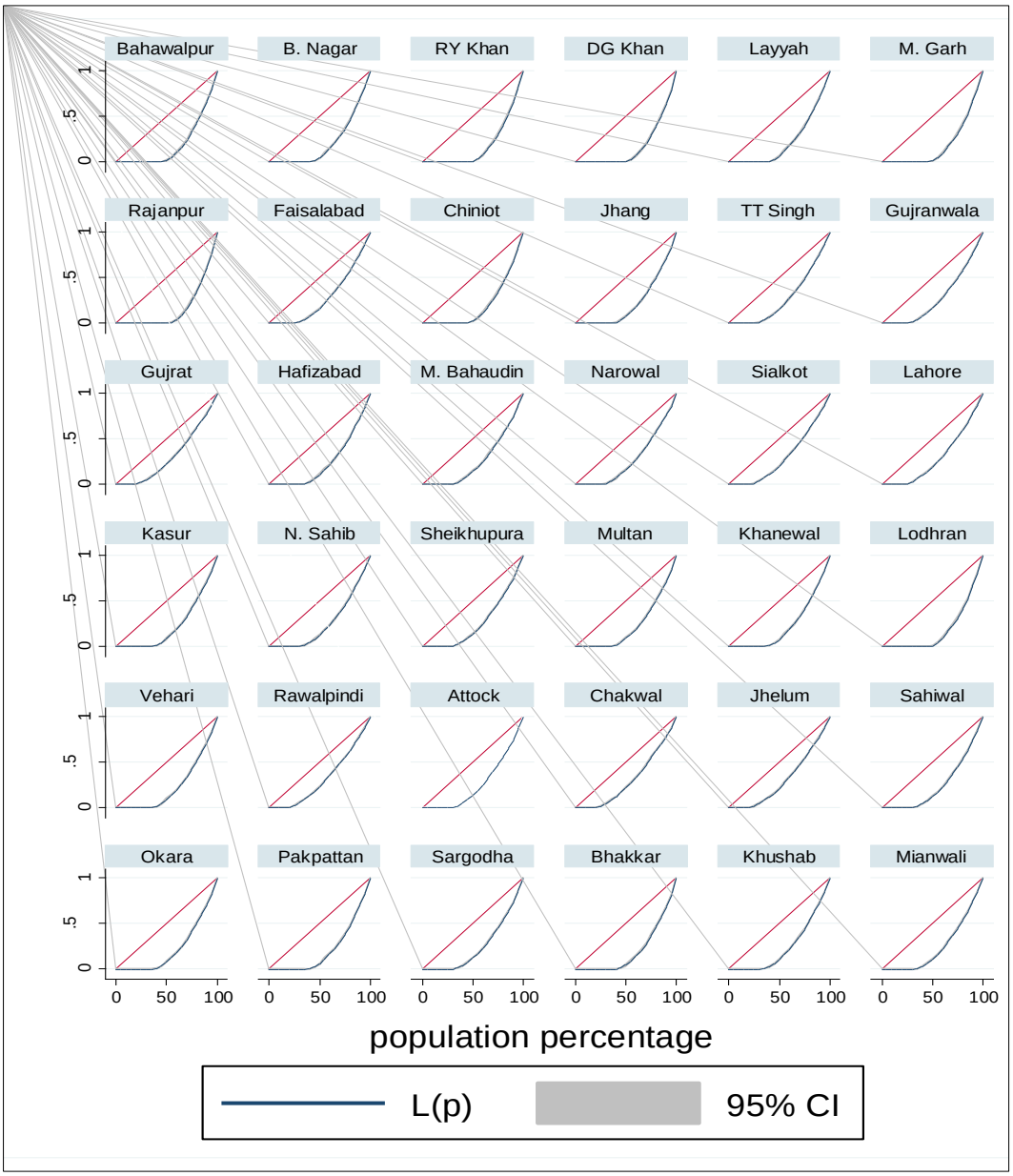
This study has used Lorenz curve, Gini-coefficient, Atkinson index and Generalised entropy class to explain the education disparities in Punjab. All disparity measures has produced similar districts' rank ordering. The figure below depicts higher female education disparities comparing to male counterpart. Similarly, a higher rural education disparities has been represented comparing to urban areas. Nearly, half females in Punjab of 10 and above age did not receive any schooling, while around 25 percent male fall in zero years of schooling category.

Figure 4: Lorenz curve for education attainment by gender and locality (Punjab)



The figure 5 below represents the district wise education disparities by Lorenz curve. Overall, figure depicts, Rawalpindi, Chakwal, Jhelum, Sialkot and Lahore are districts with least unequal distribution of educational attainment (years of schooling). On the basis of education attainment, the most deprived districts are district RY Khan, DG Khan and Rajanpur where more than half population (of 10 and above age) do have zero years of schooling. There are only two districts in Punjab, Rawalpindi and Gujrat, who have less than 15 percent population with zero years of schooling. Overall in Punjab about 30 percent population is counted with zero years of schooling.

Figure 5 (a): District wise Lorenz curve for education attainment in Punjab

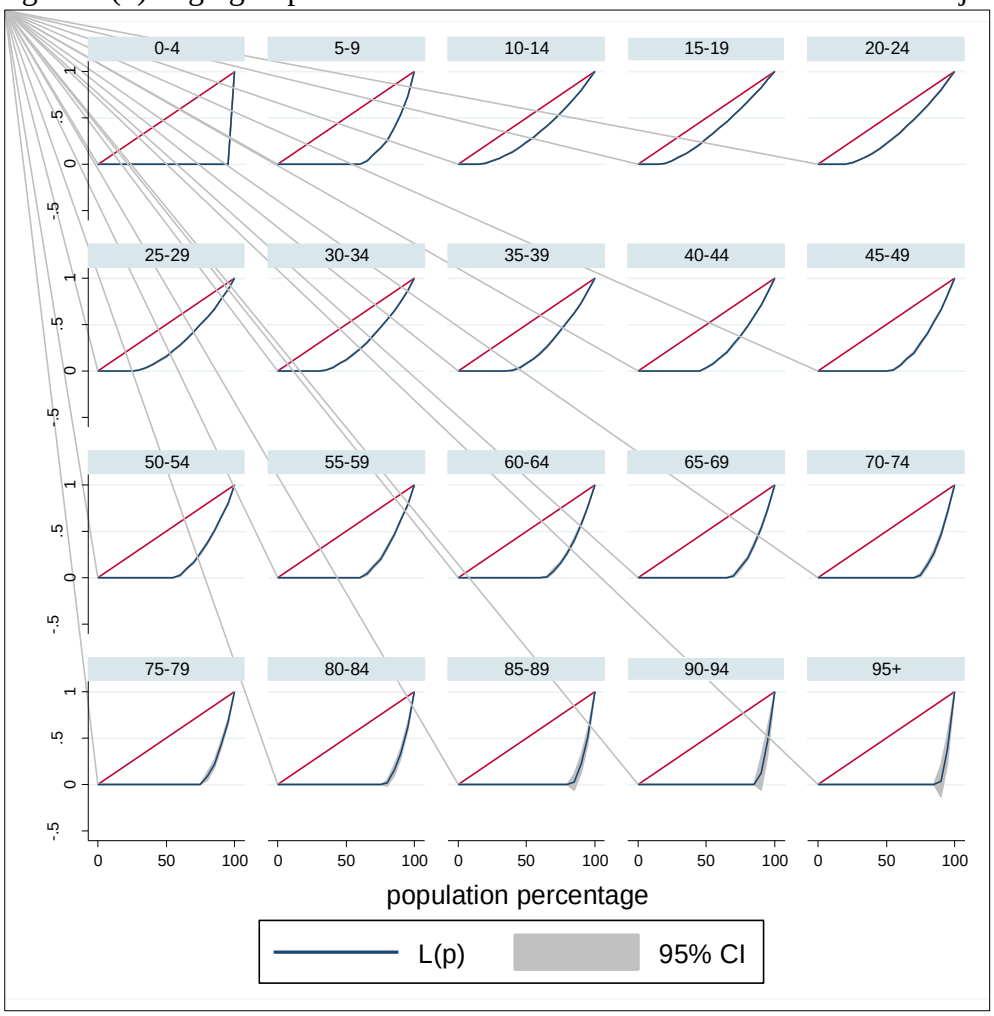


In this study, our purpose is to find either access to education is equally available for any two randomly selected households within each district, and overall in Punjab or not. Gini-coefficient is expected to be zero if every household in Punjab (as per survey data) is receiving equal level of education attainment. The lower value of Gini means, among the households, educational attainment is more equal, that indicates the social justice in society. The more intuitive interpretation of widely used inequality measure, the Gini-coefficient is presented by Raskall and Matheson (1992) as; if there are two households selected randomly from the sampled population distribution and the difference between their education attainment is expressed as a proportion of mean education, then this difference of their education achievement turns to double (twice) the value of Gini-coefficient (Gordon and Spicker, 1999).

The table 2 shows district wise Gini-coefficient of Punjab, top five districts with highest education disparities include Rajanpur, DG Khan, RY Khan, Muzaffargarh, and Bahawalpur respectively. While five districts with lowest education inequality include Chakwal, Rawalpindi, Gujrat, Jehlum and Sialkot respectively. Similar results are explained by figure 5 with help of Lorenz curves of each district. Figure 5 also illustrates age-group wise education disparities in

Punjab. As expected, age-group 10-24 has lowest inequalities and that is hope for less unequal society in future.

Figure 5 (b): Age-group wise Lorenz curve for education attainment in Punjab



The table below represents results of the disparity measures; Atkinson's index, Gini-coefficient, and generalised entropy class. The Atkinson's Index lies between zero and one, a higher value of Atkinson's indicates a higher degree of inequality in distribution of education. The table 3 presents results of Atkinson' by supposing ε (0.5, 1 and 2), where higher inequality aversion parameter takes more and more account of deprived persons, the Atkinson's index rises. When ε =2, mean deprived population is more in focus, the district RY Khan is ranked with highest education disparities followed by Bahawalpur and Rajanpur. While at parameter of aversion district Chakwal is observed having least unequal distribution of education followed by Rawalpindi and Sialkot. The Atkinson's Index increased quickly for even least unequal district Chakwal from 0.04 to 0.1 to 0.26 when ε is increased from 0.5 to 1 to 2 respectively. This quick rise in Atkinson's index with a rise in value of ε depicts that many people/households from the population that are closer to zero level of education are influencing the value of Atkinson's index. The more rapid rise in Atkinson's value with increasing aversion parameter has been observed for most unequal districts.

The value of Generalized Entropy (GE) lies between zero and infinity or one (if normalized), and zero means perfect equality of distribution. Another important characteristics of this measure is that it allows the researcher to choose any value of alpha (a parameter that assigns a weights). Where smaller values of alpha, the index would be more sensitive towards changes in the lower tail of the particular education distribution and higher value of alpha means the measure would be more sensitive to the changes at upper tail of distribution (Atkinson and

Bourguignon, 2015). The results, table below, explain highest unequal distribution of education is in district RY Khan followed by Rajanpur and Bahawalpur, these results are consistent with Atkinson's results. With any of the value of alpha, on the basis of education most unequally distributed districts in each of divisions are as follow; district Rajanpur from division DG Khan, RY Khan from Bahawalpur, Bakhar from Sargodha, Lodhran from Multan, Chiniot from Faisalabad, Kasur from Lahore, Hafizabad from Gujranwala, and Attock from Rawalpindi. More interesting results are follow; (1) all four districts of division Rawalpindi lies among the six least unequally distributed districts of Punjab, so in Punjab the division Rawalpindi is outperformer in providing equal education access to its residents, thus it is expected it would be more easily catching the SDGs 2030 targets equality, (2) out of 7 most unequally distributed districts 6 belongs to division DG Khan and Bahawalpur, so this disparity in education attainment is very much geographical and specific to government administrated unit. The biggest challenge for Punjab, in the view of achieving sustainable development goals, is to handle and provide equal education opportunities in these most deprived divisions. The detailed analysis of major determinants education disparities by taking the districts and division into account may be presented in upcoming section. In conclusion, all the disparity measures applied (Atkinson's index, Generalized Entropy and Gini-coefficient) for measurement of education disparities have suggested approximately similar district rank ordering; that is evident of reliability of the results of this research study.

Table 2: District wise education disparities (Atkinson, Gini and GE)

	Atkinson Index[10]			**Generalized Entropy [11]**				
District	**A(0.5)**	**A(1)**	**A(2)**	**GE(-1)**	**Mean log deviation GE(0)**	**Theil Index GE(1)**	**Coefficient of variation GE(2)**	**Gini-[12]Coefficient**
Chakwal	0.04	0.10	0.26	0.18	0.10	0.08	0.07	0.21
Rawalpindi	0.05	0.12	0.35	0.27	0.13	0.09	0.08	0.23
Gujrat	0.06	0.13	0.37	0.29	0.14	0.10	0.09	0.24
Jhelum	0.06	0.13	0.40	0.33	0.14	0.10	0.09	0.25
Sialkot	0.06	0.13	0.35	0.27	0.14	0.10	0.09	0.25
Lahore	0.07	0.17	0.50	0.49	0.19	0.13	0.11	0.27
Attock	0.07	0.17	0.43	0.38	0.18	0.13	0.12	0.28
Gujranwala	0.08	0.18	0.50	0.50	0.20	0.14	0.12	0.28
Narowal	0.08	0.17	0.45	0.40	0.19	0.14	0.13	0.29
Faisalabad	0.08	0.20	0.54	0.60	0.22	0.15	0.13	0.29
TT Singh	0.09	0.22	0.57	0.67	0.24	0.16	0.14	0.30
Mianwali	0.09	0.20	0.50	0.50	0.23	0.16	0.15	0.31
M. Bahaudin	0.10	0.22	0.53	0.57	0.25	0.17	0.15	0.32
Khushab	0.09	0.22	0.57	0.66	0.24	0.17	0.15	0.32
Sheikhupura	0.10	0.24	0.63	0.84	0.27	0.18	0.15	0.32
Hafizabad	0.10	0.22	0.54	0.60	0.24	0.17	0.16	0.32
N. Sahib	0.10	0.23	0.56	0.63	0.27	0.19	0.17	0.33
Sargodha	0.10	0.23	0.54	0.60	0.26	0.19	0.17	0.33
Sahiwal	0.12	0.27	0.61	0.79	0.31	0.21	0.18	0.35

10 Atkinson indices, A(e), where e > 0 is the inequality aversion parameter (An Atkinson Index for any inequality aversion parameter can be derived from a generalized entropy index under the restriction that e=1-apha, i.e an Atkinson index with high inequality aversion is derived from a GE index with small alpha. The inequality aversion parameter quantifies the amount of social utility that is assumed to be gained from complete redistribution. .

11 Generalized Entropy indices GE(a), where a = difference sensitivity parameter (the large alpha, the index is especially sensitive to the existence of higher education, whereas for small alpha, the index is especially sensitive to the existence of lower education.

12 Gini-coefficient represents lowest education inequality in district Chakwal and Rawalpindi, while highest in district Rajanpur and DG Khan (if we choose any two person at random from Chakwal, the expected difference between these two persons' education would be 42 percent on average).

Jhang	0.13	0.28	0.62	0.81	0.33	0.23	0.21	0.37
Kasur	0.14	0.31	0.68	1.04	0.37	0.24	0.21	0.37
Vehari	0.14	0.31	0.66	0.96	0.37	0.24	0.21	0.38
Multan	0.15	0.33	0.69	1.10	0.40	0.26	0.23	0.39
Khanewal	0.14	0.31	0.65	0.92	0.37	0.25	0.23	0.39
Layyah	0.14	0.31	0.64	0.90	0.37	0.26	0.24	0.39
Pakpattan	0.14	0.31	0.64	0.88	0.37	0.26	0.24	0.40
Okara	0.15	0.32	0.67	1.00	0.39	0.26	0.24	0.40
Chiniot	0.15	0.33	0.68	1.06	0.40	0.27	0.25	0.40
Bhakkar	0.16	0.34	0.69	1.13	0.42	0.28	0.26	0.41
B. Nagar	0.17	0.37	0.71	1.24	0.46	0.30	0.26	0.42
Lodhran	0.18	0.37	0.70	1.18	0.47	0.32	0.30	0.44
Bahawalpur	0.20	0.42	0.75	1.49	0.54	0.35	0.33	0.46
M. Garh	0.20	0.41	0.73	1.34	0.53	0.36	0.34	0.46
RY Khan	0.22	0.45	0.76	1.62	0.60	0.40	0.38	0.48
DG Khan	0.21	0.44	0.74	1.44	0.57	0.40	0.39	0.49
Rajanpur	0.23	0.46	0.75	1.50	0.61	0.43	0.44	0.51

Educational attainment, income and education disparities: An Empirical Analysis

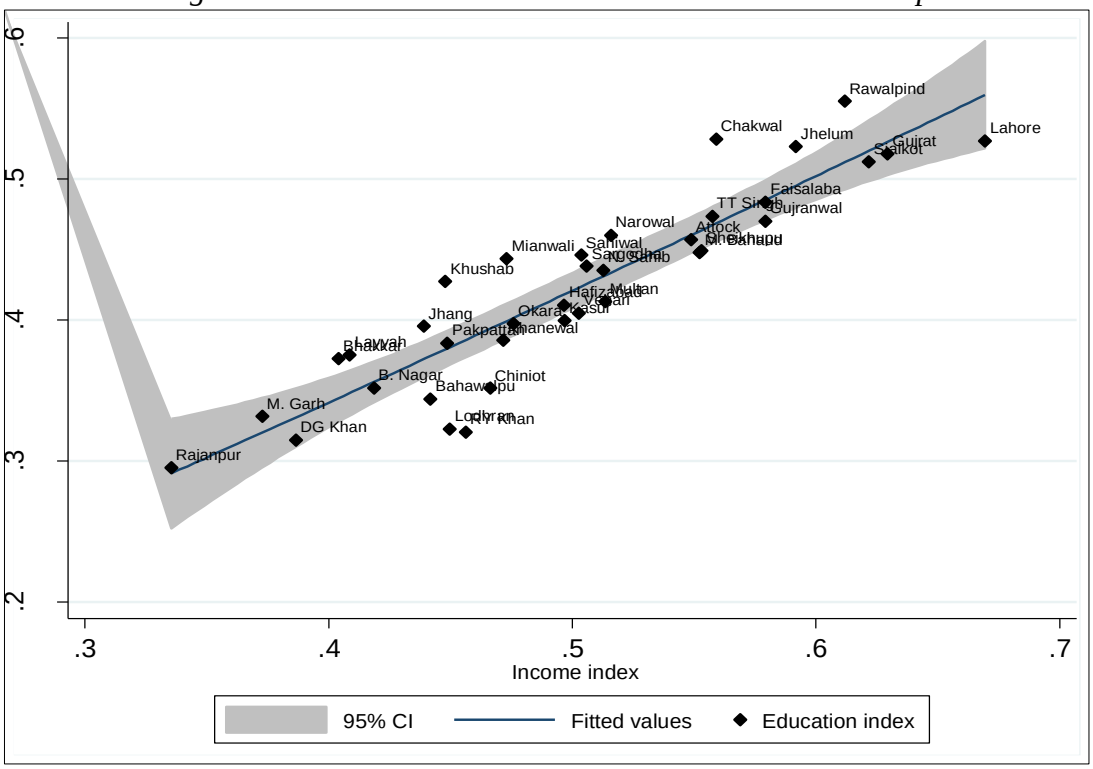
Education index is weighted average of four indicators with equal weights; including mean years of schooling, expected years of schooling, reading ability, and writing ability. Further, we have calculated education Gini, income index, gender gap (difference between female to male education), and rural to urban education differences. Moreover, we have tried to investigate relationship of these variables by answering the following questions;

1. Does there exist any association between income and education attainment? Where income index is used as proxy for income and education attainment is measured by education index.
2. What is association between educational attainment and education inequalities? Again educational attainment is measured by education index and Gini-coefficient is used to explain education inequalities.
3. Is there any association between income index and educational disparities (education Gini)?
4. Does there exit any relationship between income disparities and education disparities? Where education Gini is used for education disparities and income Gini for income disparities
5. Is there any relationship between education index and gender inequalities? Here, educational attainment female to male ratio is used as proxy for gender inequalities.
6. What is relationship between gender gap and overall educational inequalities?
7. Is there any association between education index and rural-urban education gap?
8. What is relationship between rural-urban education gap and education inequalities (education Gini)?
9. Is there any link between income Gini and rural-urban education gap?

Does there exit any association between income and education attainment? In general, a good correlation is expected between income and education indices but important is how strong relationship exists between both. Moreover, instead of current income, past income or permanent income would be good contributing measure to affect the level of education achievements. The figure 6 presents a linear association, offer nearly perfect fit, between education and income indices, here income mean permanent income. This high correlation between education and income illustrates, the households with higher ownership of assets and have better living standard (income index is computed by using asset index approach, where 22 items (asset) are selected by using Item Response Theory-IRT and equal weights are assigned for final calculation of

income index) are expected to have higher educational attainment. This reflects the calculation of education index is highly reliable.

Figure 6: Income and education indices' linear relationship



How income and education inequalities (Gini) explain the education attainment? We run simple Ordinary Least Square (OLS) regression of education index on income index and education Gini-index.

Table 3: OLS results (Education by income and education Gini)

VARIABLES	Model (1)
Income index	0.233***
	(0.0601)
Education inequality (Gini)	-0.648***
	(0.0593)
Constant	0.532***
	(0.0493)
Observations	36
R-squared	0.963

Standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Table 3 below explains the very striking results of the regression analysis, it describes nearly 96 percent variation in education is explained by income index and education Gini. The positive sign for income index represents that a rise in income (asset ownership) of household would cause to increase education attainment in Punjab. While negative sign for education disparities indicates inverse relationship between education inequalities and educational attainment. Moreover, the magnitude of negative education inequality coefficient is stronger than the positive coefficient of income index. As we examine cross-district pattern of education distribution, we found that education disparity (education Gini) decreases as average education attainment (education index) rises (see figure 7).

Figure 7: An Association between education attainment and education disparities in Punjab

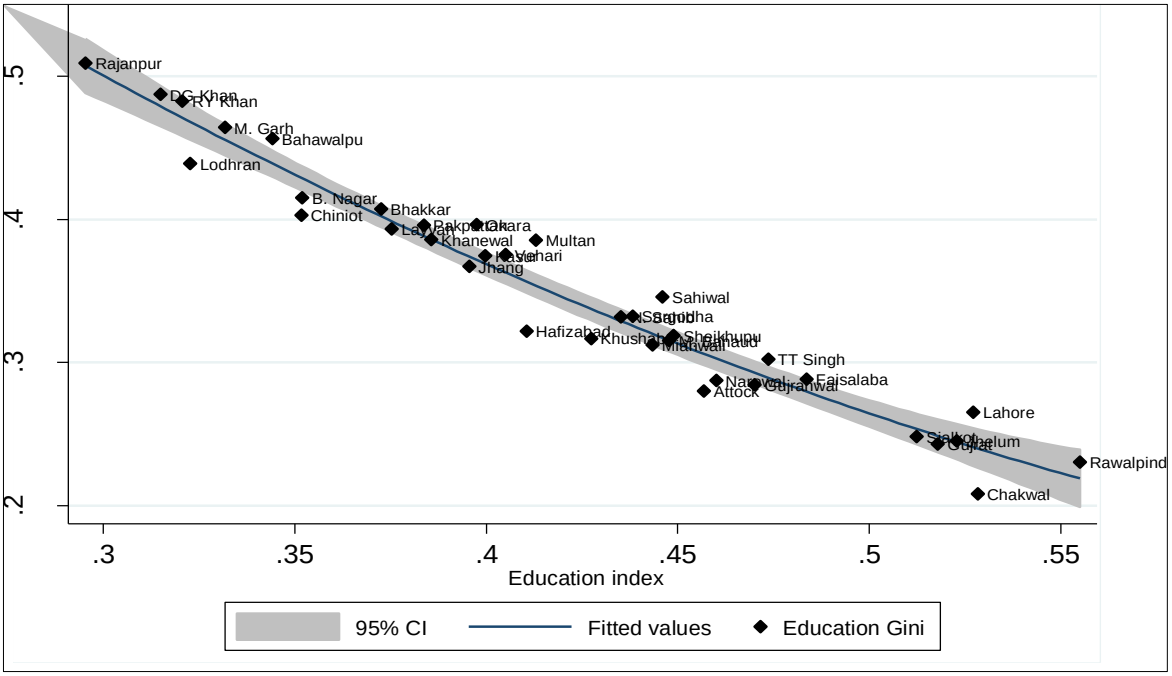
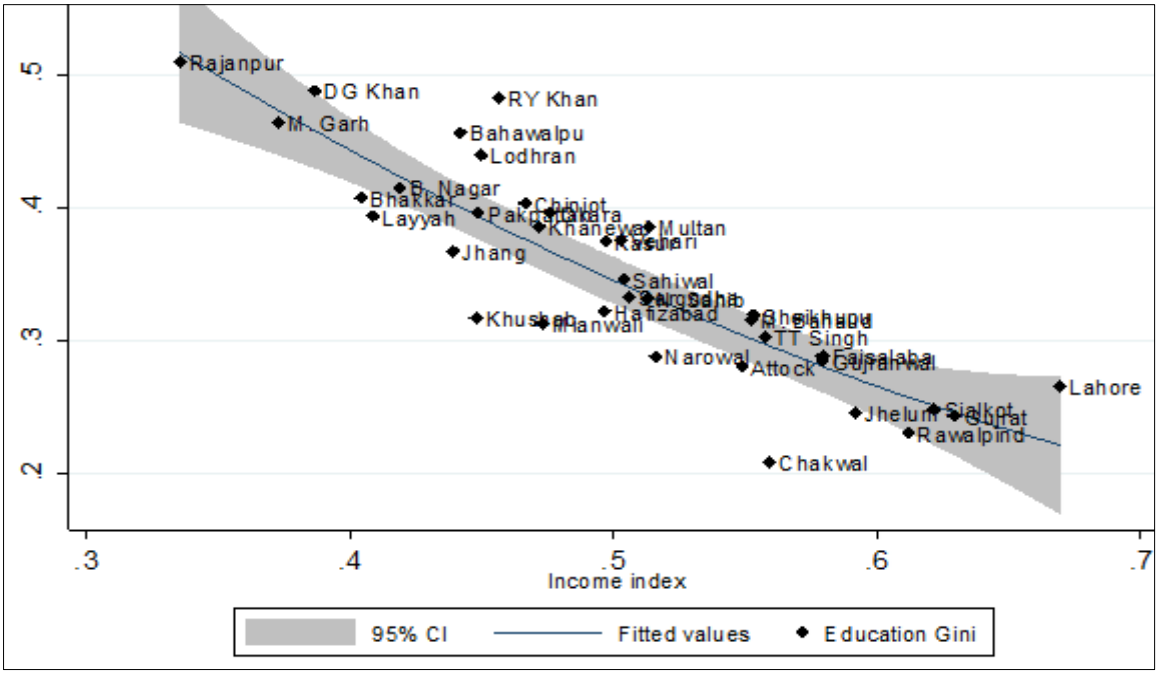


Figure 8: An Association between education Gini and income index in Punjab

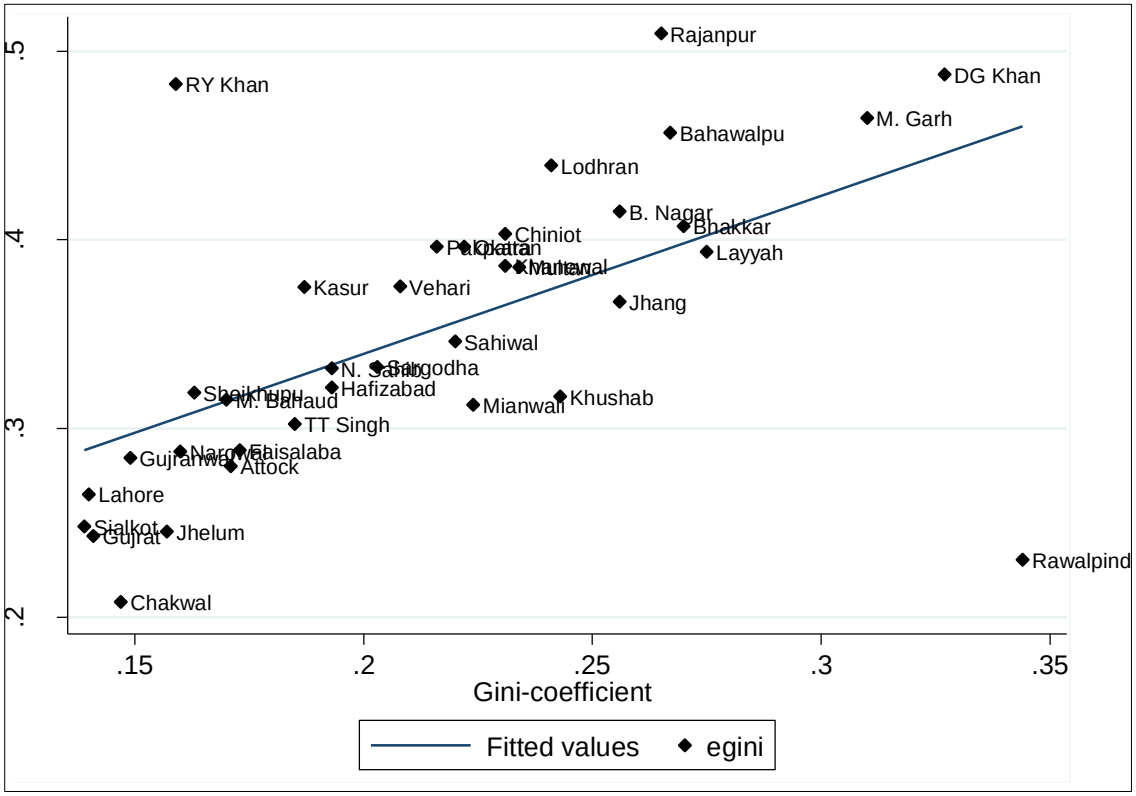


This denotes that the districts with greater average education index are most likely to attain a more fair (less unequal) education than those districts with lower average education index. Chakwal, Rawalpindi, Jhelum and Lahore stand with highest education achievements and lowest education inequality while district Rajanpur, DG Khan, RY Khan and Muzaffargarh stand at bottom in education attainments and at top education inequalities.

Does there exit any relationship between income disparities and education disparities? Figure 9 explains a positive association between income inequalities and education disparities. The districts with lowest inequalities are also evident of lowest education disparities like district Chakwal, Sialkot, Gujrat and Lahore, while districts with higher income inequalities also facing severe education disparities like DG Khan, and Muzaffargarh etc. The disparity situation is more

interesting in case of Rawalpindi and RY Khan, the case of Rawalpindi can be discussed with reference to Chakwal, the district Chakwal and Rawalpindi both are experiencing lower educational inequalities but among the equally distributed education groups income disparities very much higher. This explanation is logical because income inequalities mong the poor part of society in Rawalpindi is higher because of big city but educational access is more equal for poor and richer even.

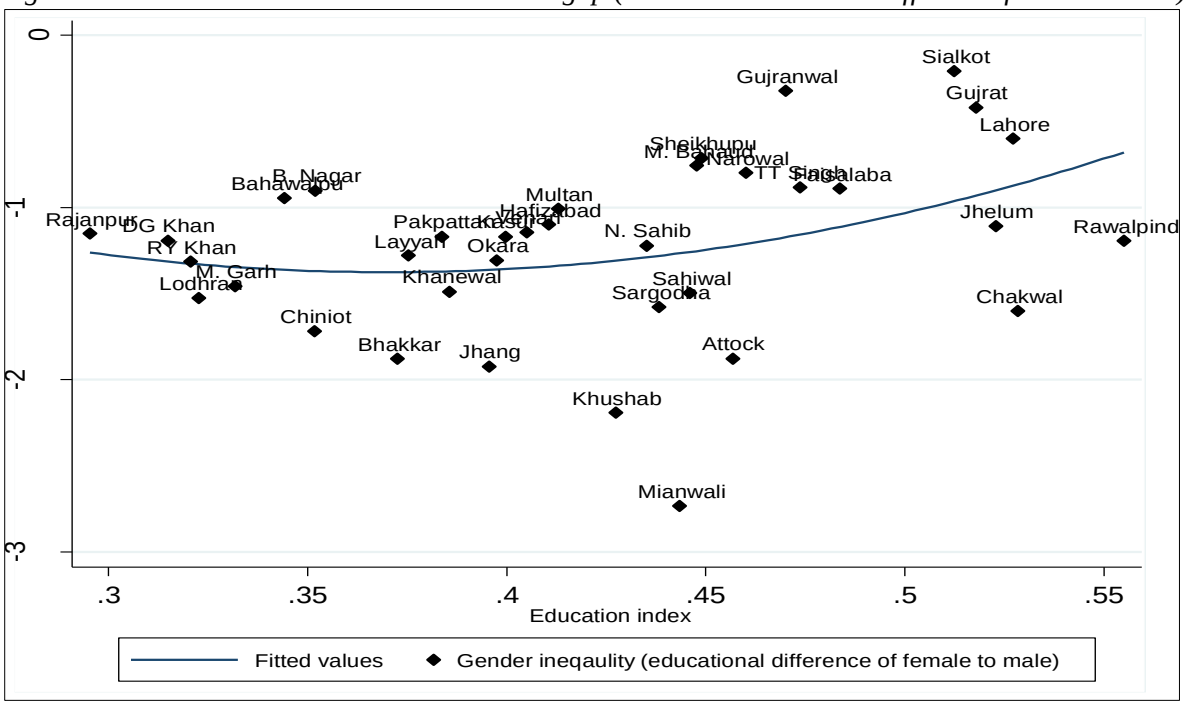
Figure 9: Association between income inequalities and education disparities



U-shaped and Inverted U-shaped Educational Kuznets Curves

What happens to education disparities with gender differences in educational attainment? If education distribution get worse off before getting better so that means Kuznetsian tale with education distribution prevails. Because society moves from zero level of education attainment to maximum level, so in beginning with educational achievements gender gap rises and education disparities (Gini) increases after a certain level Gini start declining and reach to minimum. Figure 11 explain the association between gender gap and education disparities. In figure 11, gender gap means the difference of female to male years of schooling. By showing Gini-coefficient on vertical axis and gender gap on horizontal axis a rich pattern of educational Kuznets curve is present below. Districts with higher gender disparities are facing higher education inequalities with in society like RY Khan, M. Garh, Lodhran and Jhang. While districts with less gender related education inequalities are enjoying more equal distribution of education like Sialkot, Gujrat, Lahore, Gujranwala etc.

Figure 10: Educational attainment and Gender gap (education attainment difference female to male)



Does educational attainment reduce the gender inequality in years of schooling? Figure 10 depicts that with educational attainment in beginning gender related education disparities increases (as curve move from 0 towards -3 gender related inequalities increases) after a certain point by rising educational attainment (after the education index 0.4) gender gap reduces to approach zero level (perfect equality). In beginning by rising educational attainments gender gap reduce slowly but after certain level of educational achievements gender disparities reduces sharply. More interesting findings are that in overall education and income relationships district Chakwal is outperformer but in gender related educational achievement its performance gone down and district Sialkot rise to top.

Figure 11: Education disparities and Gender gap (education attainment difference female to male)

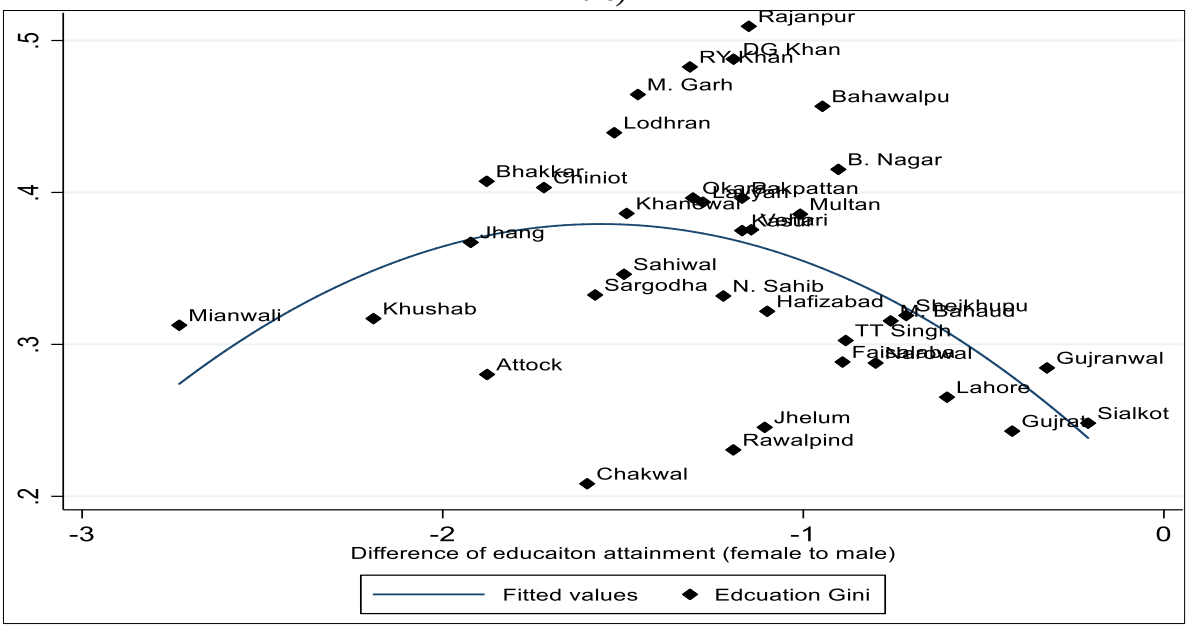
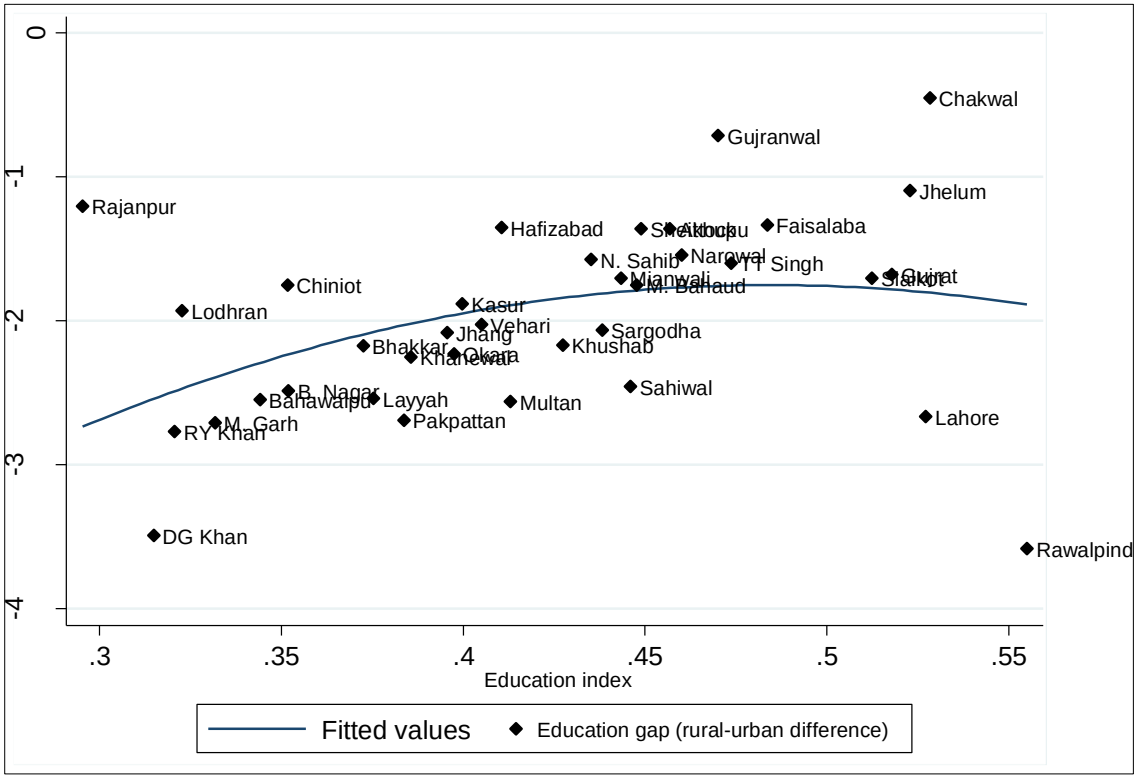
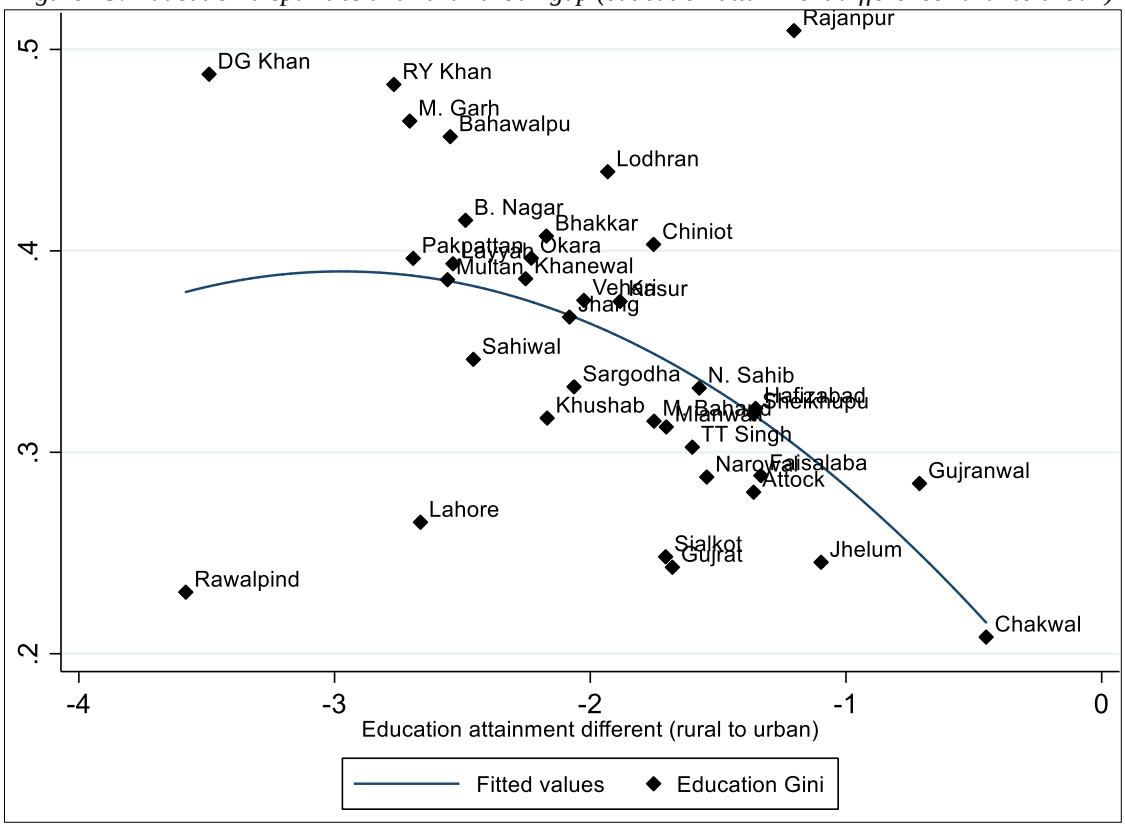


Figure 12: Educational attainment and rural-urban gap (education attainment difference rural to urban)



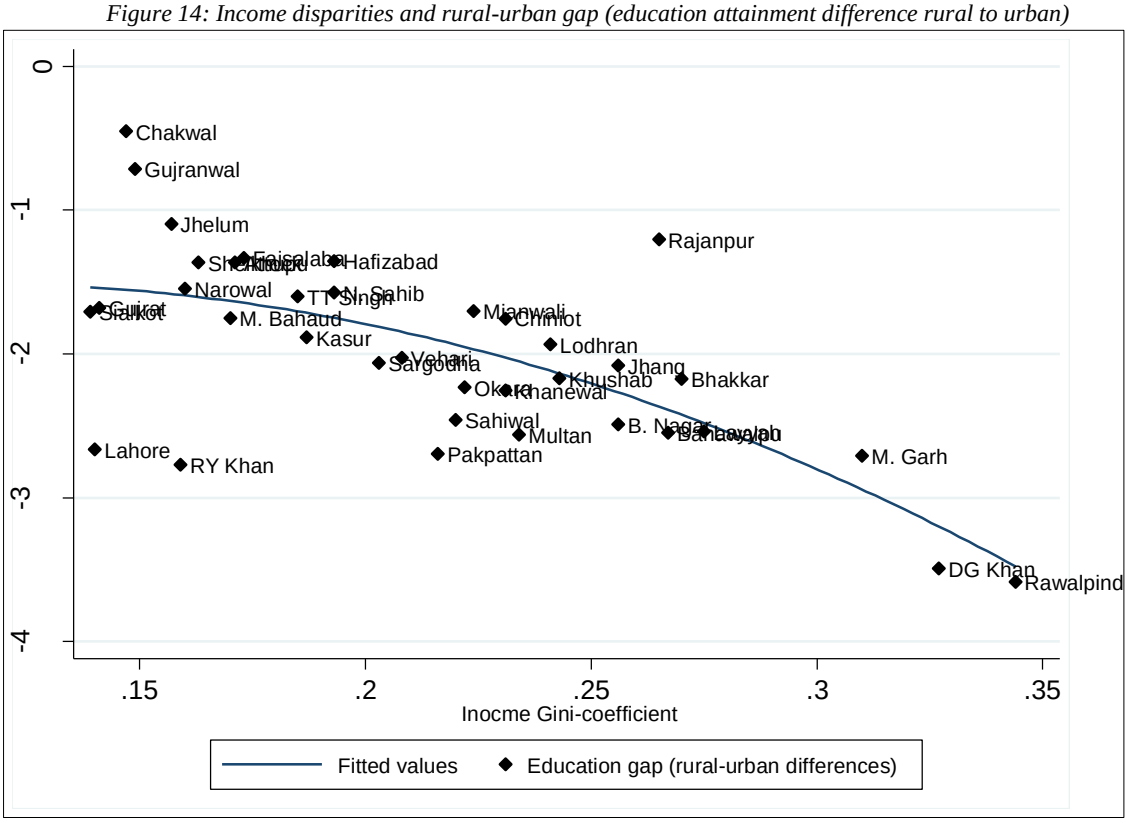
Does educational attainment reduces the rural-urban educational differences? Answer is yes, it reduces but in beginning there is share reduction in rural-urban gap but after a midpoint rural urban gap reduction become slower.

Figure 13: Education disparities and rural-urban gap (education attainment difference rural to urban)



Another interesting question is ***either rural-urban educational equality cause a reduction in overall education disparities or not?*** The figure below represents, at higher rural-urban differences educational Gini would also be higher, as rural-urban gap reduces and mover towards equality at start education disparities reduces at slower rate but after a point it decline sharply. This figure also illustrates interesting fact that district Rawalpindi and Chakwal are at bottom on overall education inequalities but both districts have entirely different rural-urban education distribution, where Chakwal performing better than Rawalpindi. Similarly, Rajanpur and DG Khan are at top with education Gini but district DG Khan has higher rural-urban gaps comparing to Rajanpur.

Figure 14: Income disparities and rural-urban gap (education attainment difference rural to urban)



Finally, ***is there any link between income Gini and rural-urban education gap?*** Figure 14 elucidates, if we read it from right to left, with reduction in income inequalities rural-urban education disparities decreases. Or districts with low income disparities like Chakwal, Gujrawala, Jhelum are enjoying lower rural-urban education gap, while districts with higher income inequalities are suffering from worse rural-urban education differences. By investigating the behaviour of the income, gender and locality related variables for education attainment and education disparities, we have found following interesting relationships among them;

- Disparities in education attainment is higher in rural areas than urban localities and female education inequalities are greater than male. Overall, gender related inequality in education is more server than rural-urban difference (see Lorenz curves for female-male and rural-urban).
- A strong linear relationship between income and education indices is observed. The districts with higher income are at the pedestal of greater educational achievements.
- There exists strong inverse relationship between education Gini and education index. This indicates that the districts with higher educational accomplishment are most likely to attain better equality in education than those with lower educational attainments.

- A strong linear negative relationship is observed between income index and education inequalities. Districts with higher income are most likely have higher potential of achieving equality in education.
- Income inequality cause educational disparities. The districts with higher income inequalities are most likely face higher education disparities.
- Educational Kuznets curve exists for education Gini and gender gap for education attainment.
- Education Kuznets curve also exist for education Gini and rural-urban education attainment differences.
- There occurs strong negative relationship between education inequality (Gini) and income index. This implies that the districts with higher income are most likely to achieve improved equality in educational attainment comparing to those districts with lower income index.

Conclusion

The education is commanding mean to strengthen the human capital, which is considered leading element for economic growth and development. It is also inevitable for poverty reduction, conflict resolution and gender equality. That's why new Sustainable Development Goals (SDGs) exclusively emphasises on the inevitability of ensuring equal access to education at all levels. The main objective of this research study is to calculate education index at household level and to estimate the education disparities for all districts of Punjab by using MICS microdata. The results of study illustrates that district Rawalpindi and Chakwal stand at top by scoring highest (in Punjab) education index value 0.78 and 0.74 respectively. On other hand, district Rajanpur and DG Khan stand lowest at the pedestal of education index value 0.41 and 0.44 respectively. Further, urban population has higher access to education facilities comparing to its rural counterpart. Surprising fact is that all the districts of division Bahwalpur, Multan, DG Khan score below the average level of educational attainment of Punjab. While all the districts Rawalpindi, Gujranwala and Lahore divisions scored above the mean value of Punjab. Excitingly, fewer districts are performing better in squeezing the gap between rural-urban educational attainments; like Chakwal and Jhelum. In general, the districts with lower education (like Rajanpur, DG Khan, RY Khan) are facing higher rural-urban education gap. The disparity measures, Lorenz curve and Gini-coefficient, explain higher female education disparities comparing to male counterpart. Locality wise, rural societies are experiencing higher education disparities comparing to urban areas. Based on Gini-coefficient, in Punjab, top five districts with highest education disparities include Rajanpur, DG Khan, RY Khan, Muzaffargarh, and Bahawalpur respectively. While the top five districts with lowest education inequalities include Chakwal, Rawalpindi, Gujrat, Jehlum and Sialkot respectively.

Concluding, all four districts of division Rawalpindi lies among the six least unequally distributed districts of Punjab, so in Punjab the division Rawalpindi is outperformer in providing equal education access to its residents, thus it is expected that Rawalpindi would be more easily catching the SDGs 2030 targets. On the other side, out of seven most unequally distributed districts 6 belongs to division DG Khan and Bahawalpur, so this disparity in education attainment is very much geographical and specific to government administrated unit, like divisions and districts. More precisely, a strong correlation is observed between income and education indices. Income (permanent) affect positively and education disparities affect inversely to the education attainment. Further, education disparity (education Gini) decreases as average education attainment (education index) rises. It has also been observed that the education disparities in low income districts is most likely to be worse than that of the high income districts of Punjab. Furthermore, a positive association between income inequalities and education disparities exists. Moreover, the districts with higher gender disparities are facing higher education inequalities with in society, while districts with less gender related education inequalities are enjoying more equal distribution of education. Likewise, at higher rural-urban differences educational Gini

would also be higher, as rural-urban gap reduces and move towards equality at start education disparities reduces at slower rate but after a point it decline sharply. The study suggests further investigation of factors of disparities with in household, across the household and across the district of Punjab. This study also concludes that education disparities are higher among Saraiki speaking districts (Southern Punjab), so Government of Punjab need to take some serious measures to investigate the socio-economic, socio-cultural and socio-political reasons and put efforts to correct accordingly.

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